This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

**Usage guidelines**

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

**About Google Book Search**

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

AW
.F987

Digitized by Google

Library
of the
University of Wisconsin

Digitized by Google

M1-L2

Digitized by Google

Digitized by Google

BULLETIN OF THE UNIVERSITY OF WISCONSIN

NO. 264

ENGINEERING SERIES, VOL. 4, NO. 6, PP. 413-500

# SELF-EXCITED POLYPHASE ASYNCHRONOUS GENERATORS

BY

LEWIS FUSSELL, M. S.

A THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY
UNIVERSITY OF WISCONSIN
1907

*Published bi-monthly by authority of law with the approval of the Regents of the University and entered as second-class matter at the post office at Madison, Wisconsin*

MADISON, WIS.
DECEMBER, 1908

PRICE 25 CENTS

Digitized by Google

# BULLETIN OF THE UNIVERSITY OF WISCONSIN

---


## COMMITTEE OF PUBLICATION

WALTER M. SMITH, *Chairman*
WILLARD G. BLEYER, *Secretary*
CHARLES M. GILLETT, *Editor*

JOHN R. COMMONS, *Economics and Political Science Series*
WILLIAM S. MARSHALL, *Science Series*
DANIEL W. MEAD, *Engineering Series*
JULIUS E. OLSON, *General Series*
EDWARD T. OWEN, *Philology and Literature Series*
WILLIAM H. LIGHTY, *University Extension Series*
WILLIAM L. WESTERMANN, *History Series*


Digitized by Google

BULLETIN OF THE UNIVERSITY OF WISCONSIN

NO. 264

ENGINEERING SERIES, VOL. 4, NO. 6, PP. 413-500

# SELF-EXCITED POLYPHASE ASYNCHRONOUS GENERATORS

BY

LEWIS FUSSELL, M. S.

A THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY
UNIVERSITY OF WISCONSIN
1907

*Published bi-monthly by authority of law with the approval of the Regents of the University and entered as second-class matter at the post office at Madison, Wisconsin*

MADISON, WIS.
DECEMBER, 1908

Digitized by Google

Digitized by Google

401225
NOV 20 1933
IWZX
F787
#3151595

# CONTENTS

| | PAGE |
|---|---|
| CHAPTER I.—Historical | 5 |
| Heyland's Diagrams | 7 |
| Heubach's Improvement | 12 |
| Ossanna's Diagram | 14 |
| Torque | 16 |
| Mechanical power | 17 |
| Efficiency | 17 |
| Slip | 18 |
| Grob's Diagram | 19 |
| Generator | 22 |
| Motor | 22 |
| Sink of energy | 22 |
| Grob's 1904 diagram | 22 |
| Torque | 24 |
| Mechanical power | 25 |
| Slip | 25 |
| Oerlikon Three Phase 20-Horse Power Motor | 26 |
| General Electric Co.'s Motor, No. 71602 | 30 |
| General Electric Co.'s Motor, No. 2577 | 33 |
| Westeras Motor | 35 |
| Brown, Boveri Motor, No. 193913 | 38 |
| Slip Scale for hypersynchronism | 45 |
| APPENDIX TO CHAPTER I | 54 |
| Kuhlmann's Diagram | 54 |
| CHAPTER II.—Self-exciting Asynchronous Generators | 58 |
| Heyland's Diagram for Compensated Motors | 59 |
| Oscillograms | 64 |
| The Machines | 76 |
| BIBLIOGRAPHY | 85 |

Digitized by Google

# PLATES


| | PAGE |
|---|---|
| Plate I. The Heyland Diagram | 8 |
| Plate II. Heyland's Improved Diagram | 10 |
| Plate III. Heubach's Diagram | 12 |
| Plate IV. Diagram of Ossanna | 16 |
| Plate V. Hugo Grob's Diagram | 21 |
| Plate VI. Diagram of Hugo Grob, 1904 | 23 |
| Plate VII. Oerlikon 20 Horse Power 3 Phase Induction Motor | 27 |
| Plate VIII. Oerlikon 20 Horse Power 3 Phase Induction Motor | 29 |
| Plate IX. Five Horse Power Induction Motor. G. E. No. 71602 | 31 |
| Plate X. Five Horse Power Induction Motor. G. E. No. 71602 | 32 |
| Plate XI. Five Horse Power Induction Motor. G. E. No. 2577 | 34 |
| Plate XII. Five Horse Power Induction Motor. G. E. No. 2577 | 36 |
| Plate XIII. Westeras 100 Horse Power 3 Phase Induction Motor | 37 |
| Plate XIV. Westeras 100 Horse Power Induction Motor | 39 |
| Plate XV. Westeras 3 Phase Induction Motor | 40 |
| Plate XVI. Grob Diagram of Brown Boveri 3 Phase Induction Motor | 41 |
| Plate XVIIa. Brown Boveri 5 Horse Power 3 Phase Induction Motor | 42 |
| Plate XVIIb. Brown Boveri Induction Motor | 43 |
| Plate XVIII. Connections | 46 |
| Plate XIX. Condenser Excitation and Pump. Back Connections | 47 |
| Plate XX. Curve I From Motor Slip Line Continued. Curve II From Corrected Slip Line | 50 |
| Plate XXI. Comparison of Slip Scales | 51 |
| Plate XXII. Karl Kuhlmann's Diagram | 55 |
| Plate XXIII. Diagram Compensated Machine | 60 |
| Plate XXIV. Heyland, July 23, 1903 | 63 |
| Plate XXV. Oscillograms | 66 |
| Plate XXVI. Voltage Determined by Capacity | 69 |
| Plate XXVII. Brown Boveri Induction Generator | 71 |
| Plate XXVIII. Brown Boveri Induction Generator | 72 |
| Plate XXIX. Winding Diagrams | 82 |
| Plate XXX. Heyland's Connections | 83 |


Digitized by Google

# SELF-EXCITED POLYPHASE ASYNCHRONOUS GENERATORS

## CHAPTER I

## HISTORICAL

The science of electromagnetism received its inception when Hans Christian Oersted, a Danish physicist, discovered on July 21, 1820, the effect produced upon a magnetic needle by the passage near it of an electric current. This discovery stimulated Andre Marie Ampere, who made public, in 1822, his laws of parallel currents; and Michael Faraday, who described first the revolution of a magnet around an electric current, and later his famous "Faraday disc" experiment.

Dominique Francois Arago[1] in 1825, made a magnetic needle follow the revolutions of a copper disc beneath it.

Then follows a long unfruitful period, until in 1879, Walter Baily[2] by the progressive shifting of the poles of two two-pole electro-magnets caused the rotation of a copper disc suspended above them. He employed two battery circuits commutated at regular intervals although he mentions that polyphase currents could well be used.

Marcel Deprez[3] in 1883 published a theorem on the true rotary magnetic field produced by two alternating fields with a quarter period phase difference.

Then follows the bitter discussion of the Tesla-Ferraris controversy, the fundamental features of which appear to be the

[1] *Annales de chemie et de physique*, **24**: 363 (1824); **28**: 375 (1825).
[2] *Phil. Mag.*, vol. 8, No. 49, p. 286.
[3] *Comptes Rendus*, 1883, p. 1193.



Digitized by Google

following: Galileo Ferraris,[4] having by 1885 independently evolved the ideas of Baily and Deprez, built a motor but failed to publish until 1888. His machine was two-phase, the second one being produced by an inductance in one circuit, the "Phase-splitter" from which Tesla obtained much credit. The model used by Ferraris developed only a watt output of 2.77 watts, and of it he says: "It is at once evident and it also results from considerations which I shall go into later on that a motor thus constructed [he used a copper cylinder] would not be of any importance as a means of transforming electrical energy." Though he regarded it as a toy, he suggested its use as a meter, and gave the idea of slip, showing also that the current in the rotating secondary varies with the slip.

Nicola Tesla applied for in 1887, and received in 1888, a great number of patents on polyphase motors, underlying which, however, was but the one idea of rotating magnetic fields. Thus, in spite of the great advance in the commercial development which is largely due to Tesla, the invention cannot be claimed for him.

Charles S. Bradley's patents of 1887, 1888 and 1889, described generators and motors of two and (1889) three phases, the rotary field being described in October, 1888. His machines were Gramme rings with equidistant taps brought out to slip rings.

The three phase Lauffen-Frankfort transmission in 1891 of about 100 H. P. over 110 miles of millimeter wire, at a frequency of 30 to 40 cycles per second and a pressure of 8,500 volts, had an average efficiency of 74 per cent. It was a very bold attempt and awakened great interest not only in the possibilities of long distance transmission but in those of polyphase motors as well, since induction motors were used to develop power at the point of delivery.

In 1895, Alexander Heyland[5] published an account of a simplified relation between the vectors of an induction motor which gave an approximation only but which tallied very satis-

[4] *Atti di Torino*, Mar. 18, 1888.
[5] *Elektrotechnische Zeitschrift*, 1895, p. 649.



Digitized by Google

factorily with the experiments he made on motors up to ten horse power. He offered no complete mathematical proof.

B. A. Behrend[6] gave independently a more rigorous proof of the relations of the vectors to a circle. Since, however, he very gracefully withdraws his claims to priority and since development has followed more particularly the lines laid down by Heyland, this discussion will take up more particularly Heyland's figures.

Julius Heubach, Karl Kuhlmann, Ossanna, Adolph Thomälin, and Hugo Grob have further developed the circle diagrams and their work will be shown later.

## Heyland's Diagrams

Heyland realized the accurate relations of primary to secondary current. (See Plate I.) Here AC′ represents the primary current, $I_1$; AC represents the magnetizing current, $I_m$; and c′C′ represents the secondary current, $I_2$. For accuracy the length c′C′ must be determined as the length of a line through C between its intersections with the semi-circles CC′D and Cc′A. The pressure line is AE.

Heyland neglects primary resistance in his electrical calculations and sets $I_1 = I_2$. This is for purposes of simplification and is not such a violent assumption as might appear at first sight, since CD is always very large as compared with AC.

"This gives us the law: In induction motors the relations between pressure and phase displacement may be represented by a vector diagram in which the vector, changing with the load which represents the current, is defined, in that its free terminus moves upon a circle whose position is defined through the relation

$$\frac{A\,D}{A\,C} = \frac{\text{Magnetic Resistance Strayfield}}{\text{Magnetic Resistance Rotorfield}}.$$

Plate I, shows AB the pressure line; AC the magnetizing current; CC° the energy component, their resultant AC° being the no-load current. The length CC° represents "The constant

[6] *Ibid.*, Feb. 1896.



Digitized by Google

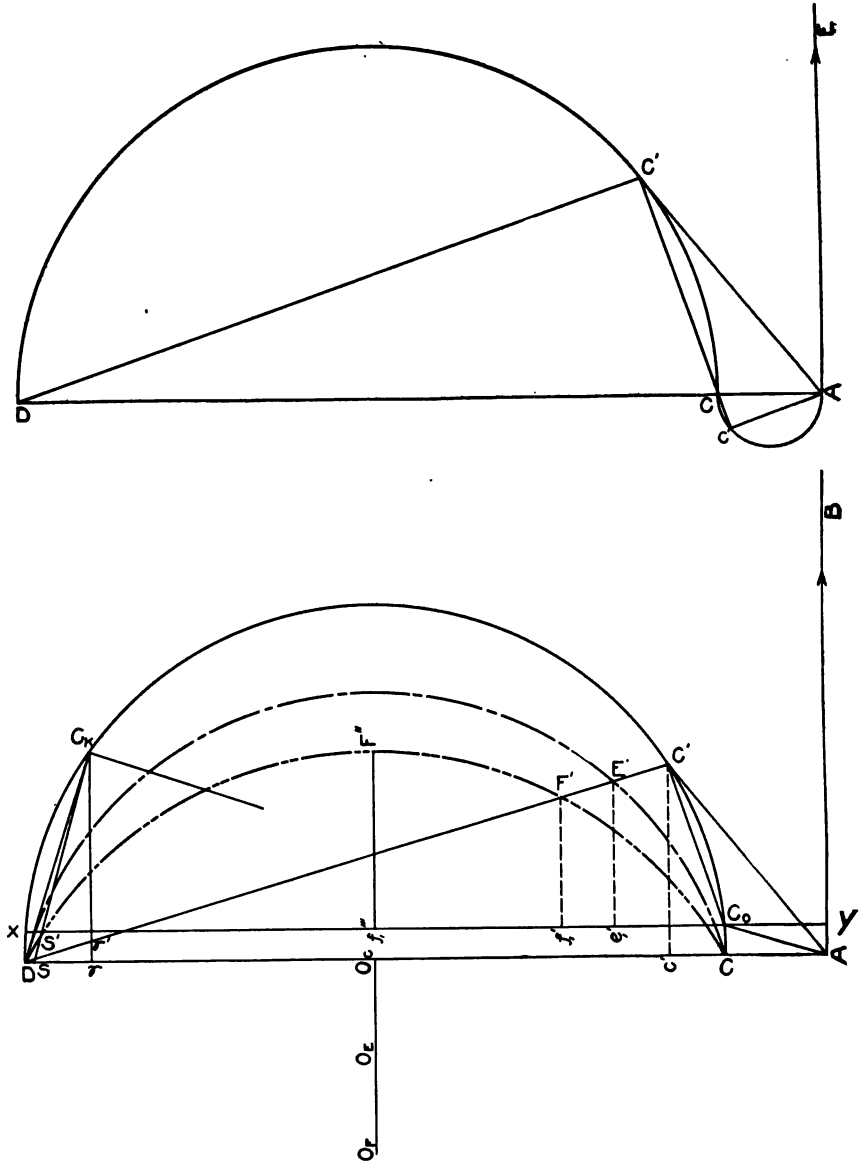



PLATE I.—THE HEYLAND DIAGRAM.

[420]

Digitized by Google

friction and iron losses, and the copper loss of the magnetizing current." $I_1 = AC'$; $I_2 = C°C'$; the angle of phase displacement is the angle $BAC'$.

Draw $DC'$, subtract along this line a length proportional to $I_1R_1$, giving the point $E'$; and from $DE'$ subtract $I_2R_2$ giving $F'$. Draw circles through $DE'C$ and $DF'C$ with centers on a perpendicular dropped from $O_c$.

A perpendicular let fall from $C'$ at any load represents electrical input, since in any triangle with constant base the area is proportional to the altitude.

Primary stray flux really $C'A$ is assumed by Heyland to be equal to $CC'$ which is the case when $AC$ becomes zero. In the triangle $DCC'$, therefore, the electric input varies as the area, since it is equal to rotor flux times primary stray flux. Since the base $CD$ is constant, the electric input varies as the altitude of the triangle $DCC'$.

Similarly, perpendiculars dropped from $E'$ and $F'$ represent torque and power output, but the constant losses $CC°$ must be subtracted, giving torque equal to $E'e'$; power equal to $F'f'$.

The maximum power output does not coincide with the maximum power input nor with the maximum current in primary nor secondary.

As $C'$ swings around counter-clockwise $E'$ and $F'$ also revolve and the maximum power output occurs when $F'$ reaches the perpendicular through $O_c$. Further increase in current decreases the output until the output becomes zero, when $F'$ and $D$ coincide, the motor comes to a standstill, $E'$ and $C'$ assume the positions $E^k$ and $C^k$ such that $DC^k$ is perpendicular to $O_fD$, that is, tangent to the power circle. The perpendicular from $E^k$ represents starting torque, that from $C^k$ power input while $AC^k$ and $C_oC^k$ represent $I_1$, and $I_2$ respectively.

Slip is the ratio $S = \frac{\omega_1 - \omega_2}{\omega_1}$ where $\omega_1$ is $2\pi$ times the primary frequency in cycles per second, and $\omega_2$ corresponds for the secondary frequency. Since the frequency of the secondary depends upon its speed of rotation, at standstill $\omega_2 = 0$ and $S = 100$ per cent.



Digitized by Google

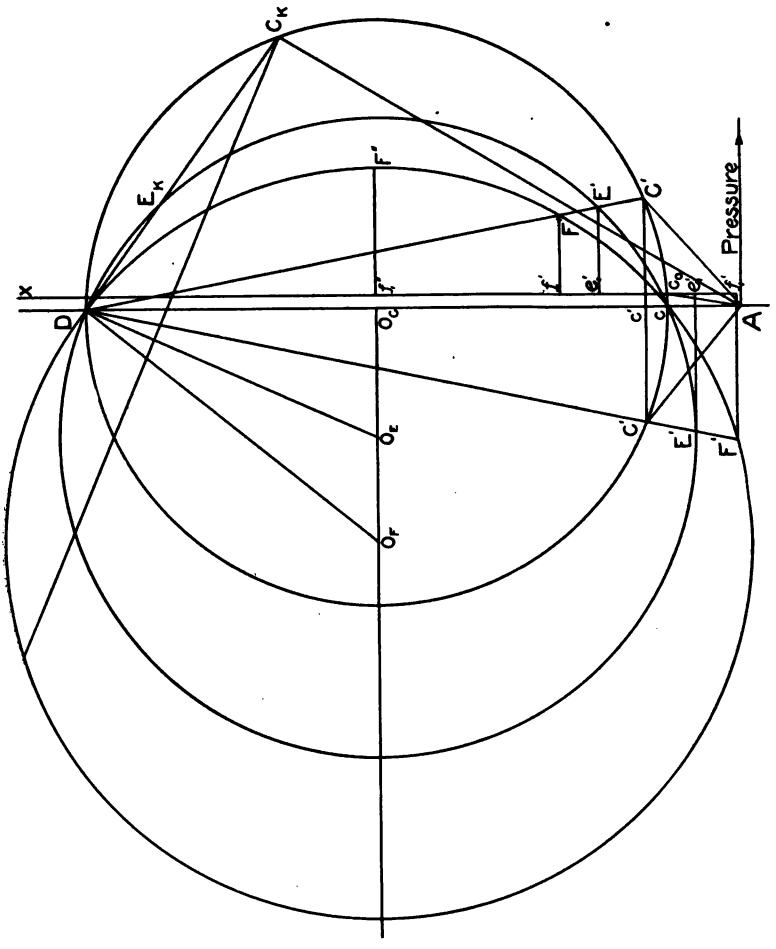



PLATE II.—HEYLAND'S IMPROVED DIAGRAM.



Digitized by Google

If we lay off $SC^k$ perpendicular to $O_eD$, we may divide $SC^k$ into 100 alloquot parts, each of which represents 1% slip, which may therefore be read off directly for any load as the distance $sS'$ along $sC^k$ from the base line AD to the intersection with the line $DC'$ for that load. Approximately $Y'C^k$ represents the efficiency at the same load since $YY'$ represents the losses.

This figure is the one commonly intended when Heyland's diagram is mentioned.

The power factor cos $\phi$ is of course a maximum when $AC'$ comes nearest to AB, that is when it is tangent to the circle $O_c$

"The characteristic of the induction motor is known that, if driven above synchronism, it works as a generator[7] and delivers current in a manner analogous to the direct current shunt machine. It is interesting that the diagram gives explanation of these relations simply by completing the circles to the left of AD—." Plate II. "The electric energy delivered by the generator at the same phase angle is exactly equal to the electric energy delivered to the motor, [This is not accurate] The mechanical power becomes naturally considerably higher and indeed as is easily seen by twice the losses since now all losses are added to the electric power which were formerly subtracted from it."[8]

He plots curves of all quantities with respect to mechanical power, but since all later writers plot with respect to slip the consideration of his curves is not of the highest importance. He makes no explanation of the phenomena from $+100\%$ to $\infty$ nor from $\infty$ to $-100\%$ simply saying that from o to $+100\%$ slip the machine functions as a motor and everywhere else as a generator. Some power could conceivably be abstracted from

[7] See Danielson, *Elec. World*, 1893, **21**: 44.

[8] Ernst Danielson, Westeras, Sweden, gave in *Electrical World* for Jan. 21, 1893, a description of experiments performed by him. Having a three phase induction motor on a synchronous generator, he reversed the power and the synchronous machine ran as a motor. "This field is capable of maintaining the current, provided the conditions in the external circuit are such that the current comes in advance of the induced E. M. F. These conditions can be fulfilled by having a regular three phase generator in circuit, or by condensors. It would be impossible to have the motor act as a generator on a regular lamp circuit or on a circuit with self induction." He took no quantitative observations.



Digitized by Google

the rotating secondary, electrically, but the better conception is that from + 100% to — 100% slip through ∞ the machine acts merely as a sink of energy requiring both mechanical and electrical input and giving no output except heat.

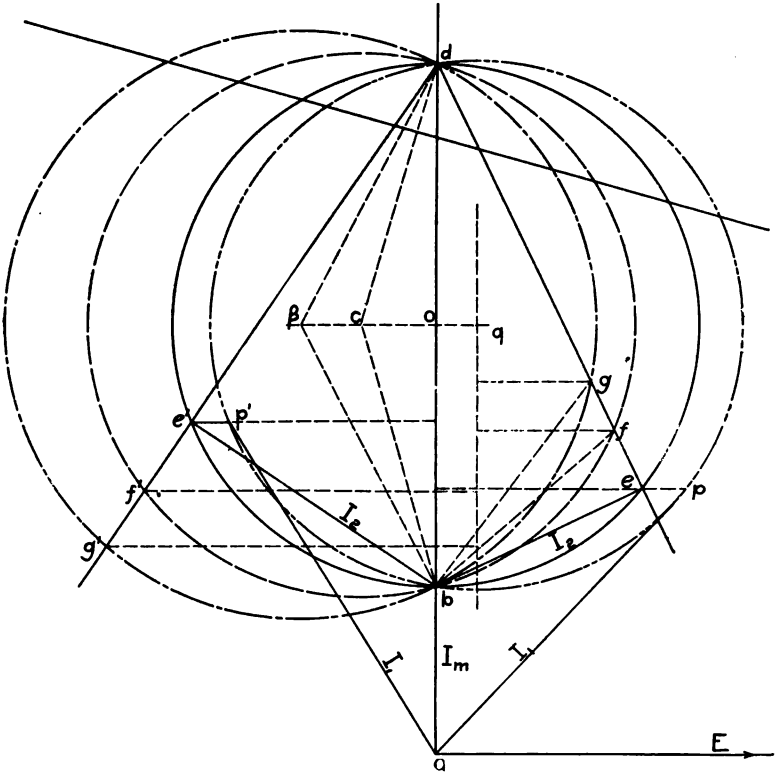



PLATE III.—HEUBACH'S DIAGRAM.

## HEUBACH'S IMPROVEMENT

Julius Heubach shows his opinion as to the magnitude of the problem very succinctly in the preface to his book *Der Drehstrommotor.*

"The motor although simple in its mechanical relations is correspondingly complicated in the electrical phenomena which



Digitized by Google

occur within the machine; yes, one can say that from the theoretical standpoint, the asynchronous (Induction) motor is the most difficult problem with which electrotechnics has to deal."

Plate III is based on the assumption that $\cos \phi_2 = 1$, and shows the condition of the induction machine from $+\infty$ to $-\infty$, the circle with center at o being the original circle used by Heyland. Heubach considers the electrical input of a motor and output of a generator as being on a circle whose center is at q so chosen that ep shall represent the losses due to hysteresis and eddy currents. The line ap, therefore, represents $I_1$; the perpendicular let fall from p to ad the electrical input; ep = hysteresis and eddy current losses; be $= I_2$; perpendicular from e to ad = electrical input minus hysteresis and eddy current losses. Now if de be drawn from d the effective flux is obtained, which is again reduced by losses due to leakage in stator and in rotor.

Laying off distances ef and fg such that $\frac{ef}{be} = R_1 = \tan < ebf$ and $\frac{fg}{bf} = R_2 = \tan < fbg$

flux values df and dg are obtained proportional to input and output of the secondary respectively. Next lay off from the line ad the $< s\ ob\ c = ebf = \alpha$ and $ob\beta = fbg = \beta$ and describe circles whose centers lie on the points of intersection of the sides $b\alpha$ and $b\beta$ with a perpendicular to ad from the point o. Then draw a straight line parallel to ad and at such a distance as to represent the losses due to bearing friction and windage. Then perpendiculars let fall from f and g on this line will give by their length values of torque and mechanical power.

Output will increase in the motor as p swings around counterclockwise until g reaches the line oq prolonged, its maximum ordinate, thereafter though $I_1$ and $I_2$ increase the output decreases steadily until, when the line de becomes tangent to the circle whose center is B, the output becomes zero and g and d coincide. This is the point of standstill or 100% slip and a line drawn at right angles to cd through this point may be divided into 100 parts, each of which represents 1% slip. For any load the slip may be read directly from the slip-line, or any propor-



Digitized by Google

tionally divided parallel line, being the length along the slip-line from ad to the intersection with the line de at that load.

Heubach says: "This figure shows the Heyland diagram for all possible conditions and it occurs peculiarly in it, that the scales for positive and negative slip are different. This peculiarity is demanded because the loss of potential in the stator winding is not considered with entire accuracy, since in this study Stator— is set equal to Rotor — current." The angle of inclination of the slip line is of course the same for positive or negative slip. On the motor side the intersection of the slip line with the solid circle is the point of 100% slip; while on the generator side the point of 100% slip is the intersection with the dot-dash circle whose center is at B.

As in the consideration of the induction machine as a motor so as a generator, the primary current $I_1$ is measured from a to the intersection of the dash-double-dot circle with a perpendicular from ad to the point of particular load. p′ to ad = electrical power output; e′p′ = loss through hysteresis and eddy currents; e′ to ad = total electrical power. Drawing de′ obtain points f′ and g′ giving the increased fields df′ and dg′ which must be developed to care for the stator and rotor losses. From f′ to the line parallel to ad gives the resistance moment or negative torque, while from g′ to the same line gives mechanical energy input. $I_2 = be'$, $I_m = ab$. As p′ revolves counter-clockwise the output rises to a maximum over point o and comes to zero when point p′ reaches d.

This is the point of — 100% slip and any further increase in speed throws the electrical power into the opposite direction, *i. e.* both mechanical and electrical power are consumed and turned into heat. This continues from — 100% to — ∞ slip and is the same condition of brake or sink of energy as is found from + 100% to + ∞ slip.

## Ossanna's Diagram

Ossanna has done a great deal toward the elucidation of induction motor problems by his diagram published in 1899.[9]

[9] *Zeitschrift für Elektrotechnik*, May, 1899. *Elektrotechnische Zeitschrift*, Aug. 23, 1900.



Digitized by Google

Of it Ossanna himself says, " . . . it is strictly accurate since in recognizing the voltage drop in the primary winding, the total primary current and not merely its watt component is considered, and further my representation of slip, torque, mechanical power and efficiency are simpler than in Heyland's diagram."

M. Breslauer says of it, "This corrected diagram is alone right and every graphical representation must be based upon it."[10]

Karl Kuhlmann says: "The Ossanna diagram in its ordinary form [see Plate IV] answers the following question: How does an asynchronous polyphase motor, with a rotor short-circuited and with a constant coefficient of self-induction, behave?"[11]

All of Ossanna's predecessors had built up diagrams for constant *back* e. m. f. and constant angle of secondary lag angle $\phi_2$, but these are not the conditions in the actual motor where we have instead constant impressed e. m. f. and constant secondary inductance $L_2$.

Taking up the figures of those who had worked before him he showed them to be inaccurate although convenient, and developed his accurate figure. In this constant impressed e. m. f., varying $I_1$, slip, and efficiency $\eta$, are the only qualities represented since the loci of the free ends of the flux and $I_2$ vectors bear no simple relation to the rest of the figure although they may be drawn for any particular load.

Ossanna's development follows:

**Take** $A_1$ = terminal pressure

$K$ = reactance of primary per phase

$\text{im} = \frac{A_1}{K}$

$\sigma = 1 - v_1 v_2$ (where $v_1$ and $v_2$ are less than unity. Stray factors)

$$\text{Then Xo} = \frac{A_1}{K}\frac{\beta}{2\alpha} \text{ and Yo} = \frac{A_1}{K}\frac{\gamma}{2\alpha}$$

$$\text{R (radius)} = \frac{A_1}{K}\frac{1}{2\alpha}\frac{1-\sigma}{\sigma}.$$

[10] *Elektrotechnische Zeitschrift*, Mar. 3, 1904.

[11] *Ibid.*, Apr. 18, 1901.



Digitized by Google

$$\text{Where } \alpha = 1 + \frac{2\pi n_1 L_1}{K} \cdot \frac{1+\sigma}{\sigma} + \frac{1}{\sigma}\,\frac{W_1^2 + 2\pi n_1 L_2^2}{K^2}$$

$$\beta = \frac{1}{\sigma}\left(1 + \sigma + 2\,\frac{2\pi n_1 L_1}{K}\right)$$

$$\gamma = \frac{1}{\sigma}\,\frac{2W_1}{K}$$

These equations determine the circle with respect to the axes.

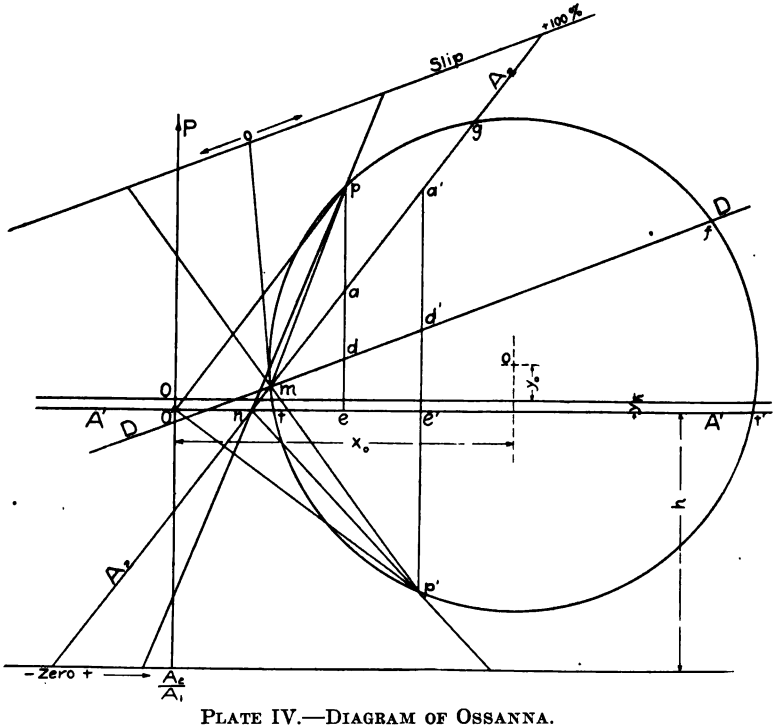



PLATE IV.—DIAGRAM OF OSSANNA.

*Torque.*—To obtain values of torque instead of drawing an extra circle as Heyland does, draw the line DD whose equation is $Z = rx - T$

(Z is ordinate, x abscissa)

$$r = \frac{w_1 \frac{\beta}{\alpha}}{K - w_1 \frac{\gamma}{\alpha}}, \text{ and } T = \frac{A_1}{K} \cdot \frac{w_1 \frac{1}{\alpha\sigma}}{K - w_1 \frac{\gamma}{\alpha}}$$



Digitized by Google

Absolute torque in meter-kilogrammes may be obtained as

$$D = \frac{p}{9.81}\frac{v_1}{v_2}\cdot\frac{1}{2\pi n_1}\left(K - w_1\frac{\gamma}{\alpha}\right)\frac{A_1}{K}(y-z)$$

$(y-z)$ = the distance pd in the figure which for any load is representative of torque.

*Mechanical Power.*—Draw for mechanical power the line $A_2A_2$ whose equation is $Z = Vx - u$

$$\text{where } V = \frac{w_1\frac{\beta}{\alpha} + w_2{}^1 v}{K - w_1\frac{\gamma}{\alpha} - w_2{}^1\mu}$$

$$u = \frac{\frac{A}{K}\cdot\frac{w_1}{\alpha\sigma} + w_2{}^1\rho}{K - w_1\frac{\gamma}{\alpha} - w_2{}^1\mu}$$

$$\mu = \frac{\gamma}{\alpha}\frac{w_1{}^2 + (K + 2\pi n_1 L_1)^2}{K^2} - 2\frac{w_1}{K}$$

$$\text{and } w_2{}^1 = (w_2 \text{ reduced to primary}) = w_2\frac{1}{1-\sigma}\left(\frac{z_1}{z_2}\right)^2\left(\frac{f_1}{f_2}\right)^2$$

$$v = \frac{\beta}{\alpha}\cdot\frac{w_1{}^2 + (K + 2\pi n_1 L_1)^2}{K^2} - 2\frac{K + 2\pi n_1 L_1}{K}$$

$$\rho = \frac{1}{\alpha\sigma}\cdot\frac{w_1{}^2 + (K + 2\pi n^1 L_1)^2}{K} - 1$$

The absolute value in watts per phase

$$A_2 = \frac{\gamma_1}{\gamma_2}\left(K - w_1\frac{\gamma}{\alpha} - w_2{}^1\mu\right)\frac{A_1}{K}(y-z)$$

Here $(y-z)$ = distance pa in the figure which is proportional to mechanical power for any load.

*Efficiency.*—Parallel to the base line and at distances yh and h draw two right lines.

yh = watt component of $I_1$ which represents the hysteresis and eddy current losses.

All electric quantities are measured from the line $A_1A$, (distant yh from the X axis) as for instance pe = watts input, $O^1p = I_1$.



Digitized by Google

Call the point of intersection of $A_1$ and $A_2$ n.

$$h = 100\,\frac{w_1\frac{\beta}{\alpha} + w_2{}^1 v}{K}$$

Lay off along the line distant from the X axis $yh + h$ parts proportional to the same scale as the line h and with the intersection with $A_2$ as zero. This will represent the relation $\frac{A_2}{A_1}$ which is efficiency $\eta$ for a motor, $\frac{1}{\eta}$ for a generator, and negative for the positions of sink of energy.

*Slip.*—For slip draw parallel to DD a line, draw tangent to m a line to intersect the above and call the common point zero slip. The point of $+100\%$ slip is at the intersection of the slip line and $A_2$ while $-100\%$ is at an equal distance the other side of the line of zero slip.[12] To obtain the per cent slip for any line prolong pm to the slip line and read. Similarly for $\%\eta$ draw pn to the efficiency line and read.

From m to t both mechanical and electrical power are required to be put in and there is no output but heat. At $t^1$ the primary current has a lag of 90° ($\cos\phi = 0$) and $I_1$ is therefore entirely wattless, this being about the only place in electrotechnics where a lag of exactly 90° can be obtained.

From t to $t^1$ the machine acts as a generator, from m to g as a motor, and from g to $t^1$ through f as a sink of energy.

Plate IV shows a motor with highly exaggerated leakage, high resistance in both primary and secondary, and large iron losses, so that all the parts and processes may be made more clear.

The greatest values of mechanical power, torque and electric power, occur at points where the perpendicular to the lines $A_2$, D, and $A_1$, respectively, have their greatest lengths.

The difficulty of determining the distances $y_o$, $X_o$, yh and h and the points m and n, the fact that secondary current and the fluxes are not represented have prevented the adoption of Ossanna's diagram to so large an extent as its accuracy would seem

[12] The point of — 100 per cent. slip should be at the point where t'm prolonged cuts the slip line, but Ossanna's figures show an error in this point.



Digitized by Google

to make probable. The necessity of converting experimental values into analytical geometry and then putting that into a figure also obscures the direct connection of machine and figure.

## GROB'S DIAGRAM

Hugo Grob describes ". . . a theoretically absolutely correct polyphase induction motor diagram which has no negligibilities other than the inevitable assumption of constant permeability of the iron; of sine form of current, pressure and fields; and farther the not quite accurate allowance for iron losses."[13]

It is a diagram of constant terminal pressure and the main circle, the locus of the free terminus of primary current vector, is the same as Ossanna's although more readily located. Heyland's approximate diagram is easy to make but not nearly accurate, while his accurate one is very complicated. Ossanna brings to bear analytical geometry and deduces equations for circle and lines which represent primary current, phase angle, output, slip, etc., but has not the fluxes nor secondary current and does require some not inconsiderable calculation.

Grob develops his diagram without using analytical geometry nor higher mathematics as follows:

On a line OZ (Plate V) lay off

$$AD = \frac{1}{T_1 + T_2} \text{ OA or more accurately}$$

$$AD = \frac{1}{T_1 + T_2 + T_1 T_2} .OA$$

where $T_1$ and $T_2$ are the usual leakage factors (here taken equal to .2). Lay off $OB : BA = T_2 : T_1$ more accurately $= (T_2 + T_1 T_2) : T_1$, erect DG perpendicular to OZ at D so that $DG : OD = T_3 : (T_1 + T_2)$ more accurately $= T_3 : (T_1 + T_2 + T_1 T_2)$ where $T_3 = \frac{I_1 R_1 \text{ at no load}}{\text{Constant terminal e. m. f.}}$

Draw OG, and a perpendicular to OZ through A cutting OG in H. On BH and HD as diameters describe circles. Draw the pressure triangle with right angle at K such that $OK : OL = T_3 : 1$.

[13] *Elektrotechnische Zeitschrift*, Jan. 24, 1901.



Digitized by Google

OL is the constant terminal pressure and is at right angles to HMD.

The larger circle, the locus of the points N, is Ossanna's, ON representing primary flux or current, NHQ representing secondary flux or current. Parallel to HMD and therefore perpendicular to OL through O draw OJE. Parallel to OHG through E draw the slip line; calling the intersection with the line AH produced zero slip and the point E — 100% divide up the line. Draw a line from + 100% slip through H to cut the circle at F. The point of ± ∞ slip is marked ∞.

A method is now available for representing each of the three quantities, torque, electrical, and mechanical power.

Bisect the angle CHG and draw $N_+HQ_+U_+$ (OQ represents flux).

$ON_+$ is $I_1$ at time of greatest torque. Perpendicular to OG at H draw HV. Parallel to OD at $U_+$ draw $U_+V$ ($U_+$ is any point on line $U_+N_+$).

With V as center and radius VH draw a circle. UW for any load = Torque. (W on the line AH) Torque is also represented by a perpendicular let fall from N on A ∞.

Let $VU = \frac{1}{2}$ maximum horse power = $3/2 \cdot \frac{I_2^2 \text{ (max)} R_2}{746}$.

Draw with center *U*, and radius *UH* a circle. Parallel to AH draw a line such that *DW* is the power lost in bearing friction and windage. Then *BD* = mechanical power.

Electric power, since the pressure is constant, can be represented as the watt component of the primary current, therefore from N drop perpendiculars on OL and the watt component OP is obtained. Electric power is also represented by perpendiculars let fall on the line JE (since JE is at right angles to OL).

Starting at —∞ with either Plate V or Plate XVIIa we find a region of sink of energy. Here $I_1 = 0$ ∞, $I_2 = \infty$ $Q_0$, torque = 0, electrical and mechanical power both absorbed. As we slow down $I_1$ and $I_2$ grow, reaching a maximum when $I_1$ passes through $M_1$ and $I_2$ is parallel to OD. Mechanical power and torque grow; electrical comes to zero at — 100% slip and thereafter becomes output. At — 100%, $I_1$ is perpendicular to OL and is pure wattless current.



Digitized by Google

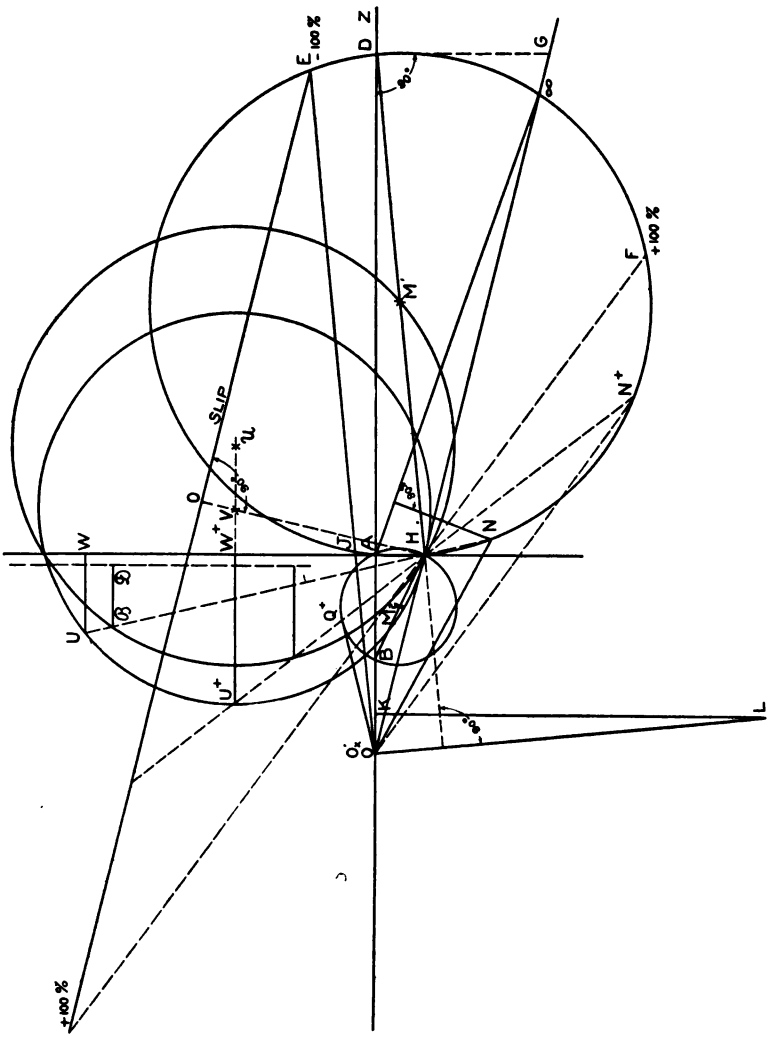



PLATE V.—HUGO GROB'S DIAGRAM.



Digitized by Google

*Generator.*—Below —100% slip all three increase, reaching their maxima in the order, mechanical power, torque, and electrical power after which all decrease.

*Sink of Energy.*—At a small negative slip ($I_1 = OJ$) electrical power becomes zero again with $I_1$ wattless. At synchronism torque becomes zero, $I_1 = OA_1$, $I_2 =$ zero. From J to H both powers are absorbed. At a small positive slip ($I_1$ at H) mechanical power becomes zero.

*Motor.*—After passing through their zero points each has a sign opposite to that with which it started. Electrical is now input, mechanical and torque, output. All rise reaching their maxima in the order, mechanical, torque, electrical power. At + 100 per cent. or standstill mechanical power becomes zero, and electrical and torque give the starting values.

*Sink of energy.*—Mechanical power reverses and at + ∞ we again strike the point of beginning with torque = o, mechanical and electrical power both consumed.

## Grob's 1904 Diagram

In 1904[14] Grob developed another diagram, not mentioning his article of 1901 except in an introductory footnote.

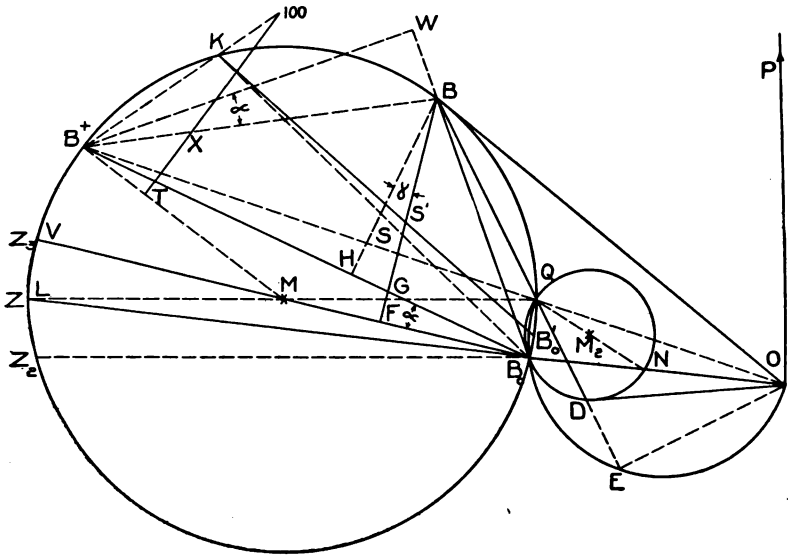
The new figure (see Plate VI) differs in that it is more accurate, represents power and torque on the main circle by lines simply located, and determines the potential line somewhat differently.

He gives the method of construction and a concrete example from data for a particular motor. This example and diagrams for four other motors will be given. The constant pressure in the new figure is chosen as the basis and perpendiculars are erected to it. The leakage factors which for the 1901 circle had to be determined in order to construct the figure are now deduced from the figure. The value $T_3$ alone need be predetermined and it is easily obtained. The lines for torque and mechanical power (as will be proven shortly) are on the circle and he has corrected the torque line, making it the distance inter-

[14] *Ibid.*, June 2 and June 9, 1904.



Digitized by Google



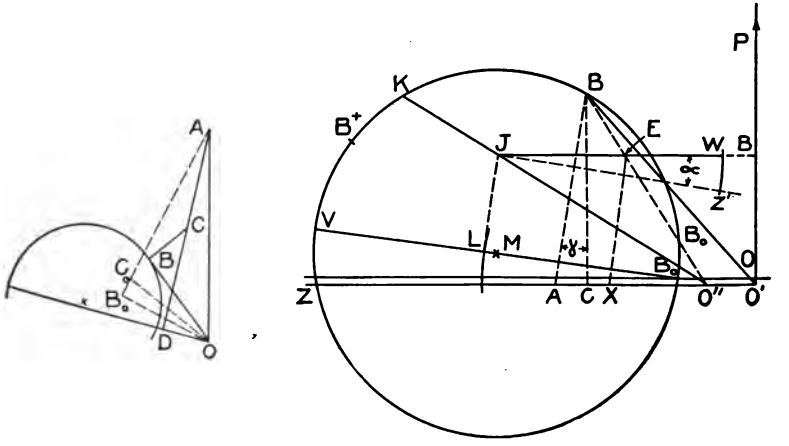




PLATE VI.—DIAGRAM OF HUGO GROB, 1904.



Digitized by Google

cepted by the line $B_0B^+$ on the perpendicular let fall from B to the line $B_0V$ instead of the length of the perpendicular to the line $B_0B^+$. Also mechanical power is obtained as the interception of the same perpendicular as for torque with the line $B_0'K$.

Total power equals the area of the triangle $ODB = DB \times OE$. (Point E moves on the semicircle over OQ.)

The triangle $BB_0D$ remains always similar to itself, peripheral angles of constant arc.

$$\frac{DB}{BB_0} = \text{constant} = \frac{LQ}{LN} \text{ (same peripheral angles)}$$

Therefore $I_2 = \text{chord } B_0B \times \frac{NL}{LQ}$

But $\frac{NL}{LQ} = \frac{OA}{AC_0}$

Therefore $DB = BB_0 \times \frac{OA}{AC_0}$

Also the arc $OEB_0$ is similar to the arc $B_0BB^+$, therefore the triangles also are similar and $OE = OB_0 \times \frac{B^+B}{B^+B_0}$

The vector $OB_0$ is the no load flux, which calls up the pressure left for developing power when ohmic and inductive drop have been deducted. This pressure is $AC_0$ perpendicular to $OB_0$.

$$OB_0 = P. \frac{AC_0}{OA} \text{ in volts.}$$

$$\text{Therefore } OE = P. \frac{AC_0}{OA} \cdot \frac{B^+B}{B^+B_0}$$

*Torque:*

$$DB. OE = B_0B. \frac{OA}{OC_0} \cdot P. \frac{AC_0}{OA} \cdot \frac{B^+B}{B^+B_0}$$

$$= P \frac{B_0B \cdot B^+B}{B^+B_0} = \frac{P}{B^+B_0} \cdot B_0B \times \frac{B^+W}{\cos \alpha} \cdot \frac{2}{2}$$

$$= \frac{P \times 2}{B^+B_0 \cos \alpha} \text{ (area of triangle } B_0BB\times)$$

$$= \frac{P \times 2}{B^+B_0} \times \frac{B+B_0 \times BH}{2} = \frac{P.BH}{\cos \alpha} = P. BG$$



Digitized by Google

In absolute measure then

$$\text{Torque} = \frac{\frac{\text{P (volts)} \times \text{BG (amps.)} \times 3}{746}}{2\pi\frac{n}{60}} \times 75 \text{ (In Kg} - \text{m)}$$

$$= 2.88 \frac{P \times BG}{n} \text{ (In Kg} - \text{m)}$$

$$\text{or} = 208 \frac{P \times BG}{n} \text{ (In lb. ft.)}$$

*Mechanical Power:* Loss in the rotor is proportional to $I_2^2$, or to $\overline{DB}^2$ or to $\overline{B_0B}^2$ or to $B_0F$, FS is proportional to the copper loss but since FG is also proportional to $B_0F$ we can take GS as a measure of the copper losses.

Then BG — GS = BS = Power developed. To subtract bearing friction and windage, lay off $B_0B_0'$ perpendicular to $B_0V$ and equal to the watt current of the no load losses. Draw $B_0'K$, and $BS'$ is the actual mechanical power available. In absolute measure mechanical power is given by

$$\text{H. P.} = \frac{\text{P (volts)} \times \text{BS}^1 \text{ (amperes)} \times 3.}{746}$$

To care for iron losses drop O to O′.

*Slip:*

$$\text{Slip} = \frac{\text{Rotor losses in heat}}{\text{Energy output of rotor}} = \frac{B_0G}{BG}$$

Draw anywhere a perpendicular to B+M, draw B+K and prolong to intersection. The triangle B+XT is similar to $B_0GB$.

$$\text{Therefore } \frac{B_0G}{GB} = \frac{XT}{TB\times} = \text{slip.}$$

Slip is measured along TX in per cent.[15]

*Efficiency:*

$$\text{Efficiency} = \eta = \frac{BS^1}{\text{OB projected on OP}} = \frac{BS^1}{BC} = \frac{BS^1}{BA}\frac{1}{\cos\alpha}, \text{ where } \alpha = \text{the angle CBA.}$$

[15] A very slight inaccuracy in the slip scale comes in for light loads from the neglect of rotor hysteresis.



Digitized by Google

Prolong $KB_0'$ to O″ the intersection with O′Z. Erect JR perpendicular to OP. Draw O″BE. Draw EX parallel to BA.

$$\text{Then } \eta = \frac{EN}{EX}\frac{1}{\cos\alpha} = \frac{EN}{O''R}\frac{1}{\cos\alpha} = \frac{JE}{JR\cos\alpha}$$

Draw JZ parallel to $B_0V$ and strike the arc ZW. Then the angle $RJZ = a$ and $JW = JR\cos a$

$$\text{Finally } \eta = \frac{JE}{JW}.$$

For simple construction, lay off $O''L = 100$ mm. along $B_0V$, erect a perpendicular to $B_0V$ cutting O″K in J. Draw JR at right angles to OP. Strike off $JW = 100$ mm.

Read $\eta$ as length JE in mm., or per cent. E is determined as the intersection of JW and the line O″B for each load.

Data, work up and curves now follow for five machines. The first is given by Grob, three are in the electrical laboratories of the University of Wisconsin, and one is derived from H. M. Hobart's book, *Electric Motors.*

### Oerlikon Twenty Horse Power Three Phase Motor

Data given by Grob. 380 volts; 1,500 rev. per. min.; 50 cycles; $R_1 = .212$ ohms; $R_2 = .0386$ ohms; Ratio of stator to rotor turns $= \frac{1}{.381}$.

No load.

$I_1 = 7.3$, $V = 380$, 50 cycles, 880 watts, 450 watts friction and windage, $\cos\phi_0 = .184$.

Short circuit. (Rotor slowly running.)

35 amperes, 50 frequency, 77 Volts, 2,050 watts, $\cos\phi_k = .437$.

$I_k$ at 380 volts $= 35 \times \frac{380}{77} = 172.5$ amperes.

Eddy current loss in the Stator.

$$\text{When } I_1 = 35,\ I_2 = \frac{I_1}{.381} = 92,\ I_2^2R_2 = 778,\ I_1^2R_1 = 980$$

Total 1,758. The wattmeter read, however, 2,050. Then, since the current density should be about the same in stator and



Digitized by Google

rotor, increase the stator. That is, eddy currents effectively increase the stator R.

$2050 - 1758 = 292$. Add $\frac{292}{2}$ to each then $I_1^2R_1 = 778 + 146 = 924$ and $R_1^1 = .212 \times \frac{924}{778} = .252$ ohms.

Short circuit $I_2^2R_2$ at 380 volts $= 980 \left(\frac{380}{77}\right)^2 = 24{,}000$ watts

watt current $= \frac{24000}{380 \times \sqrt{3}} = 36.4$ amperes.

Construct on the pressure line a power factor semi-circle of 100 mm. diameter, lay off at the proper angle and to suitable

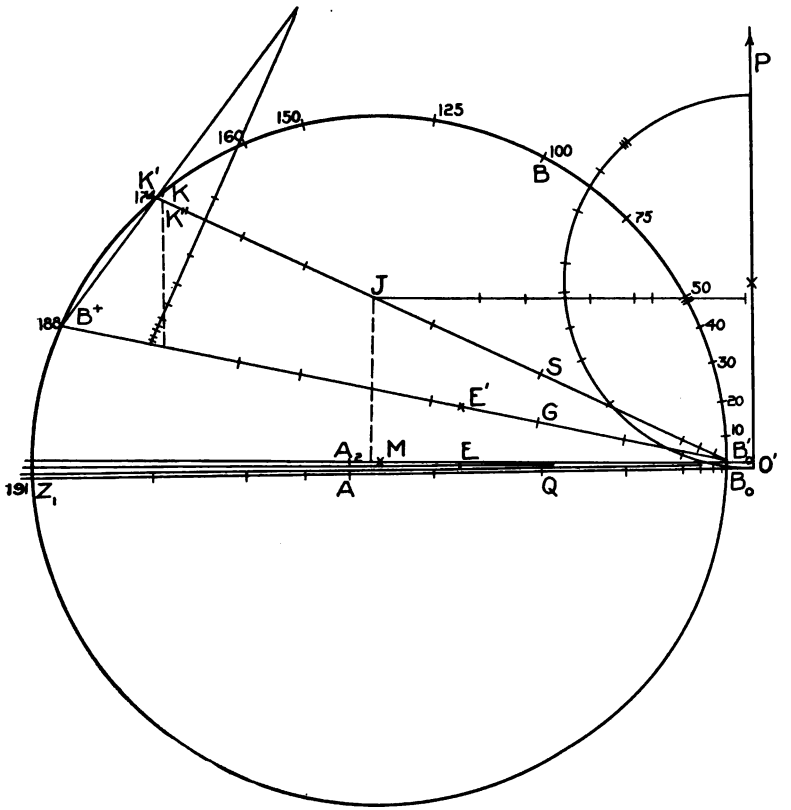



PLATE VII.—OERLIKON 20 HORSE POWER 3 PHASE INDUCTION MOTOR.



Digitized by Google

scale the short circuit and no load vectors. Subtract the watt component of the no load current $B_oB_o'$. (See Plate VII.)

Draw perpendiculars to O'P in O' and through $B_o$; draw $B_oC_o$.

$O'C_o$ = wattless component of no load current. $7.2 \times 0.252 = 1.82$

$$T_s = \frac{1.82}{\left(\frac{380}{\sqrt{3}}\right)} = .00828$$

Lay off $B_oA = 100$ mm. $A_1A_2 = .828$ mm. Construct circle with center on $B_oA_2$ which shall pass through K and $B_o$.

$$T = \frac{OB_o}{B_oL} = .0392$$

Lay off $OE = T$ and $EE_1 = T_s$

Infinite slip is the intersection with the circle of $OE_1$ prolonged.

Draw $B^+B_o$

Drop a perpendicular from K toward diameter $B_oV$ to intersection with $B_oB^+$; from this intersection backwards lay off the watt current of rotor loss at short circuit. Draw $B_o'K''$ through to intersection of line with the circle at $K^1$.

$B_oB^+$ is the torque line.

$B_o'K'$ is the mechanical power line.

$O'Z_1$ is the electrical power line.

Torque and mechanical power are the lengths for any load of the perpendicular to the diameter $B_oV$ as far from the circles as the intersections with the torque and power lines respectively. Electrical power is the perpendicular distance to the line $O'Z_1$.

To lay off the slip line, draw and prolong $B^+K'$. Perpendicular to $B^+H$ (a radius) erect TY so that the intercept between $B_oB^+$ and $B^+K'$ shall be for convenience, just 100 mm. Slip for any load is the distance TX the intercept with the line $B^+B$ for that load read directly in mm. or per cent. [See Plate VI].

Scale of efficiencies. Prolong $K'B_o'$ to O'', its intersection with $O'Z_1$. Describe an arc 100 mm. long. Erect a tangent perpendicular to the diameter $B_oV$ to intersection with the



Digitized by Google

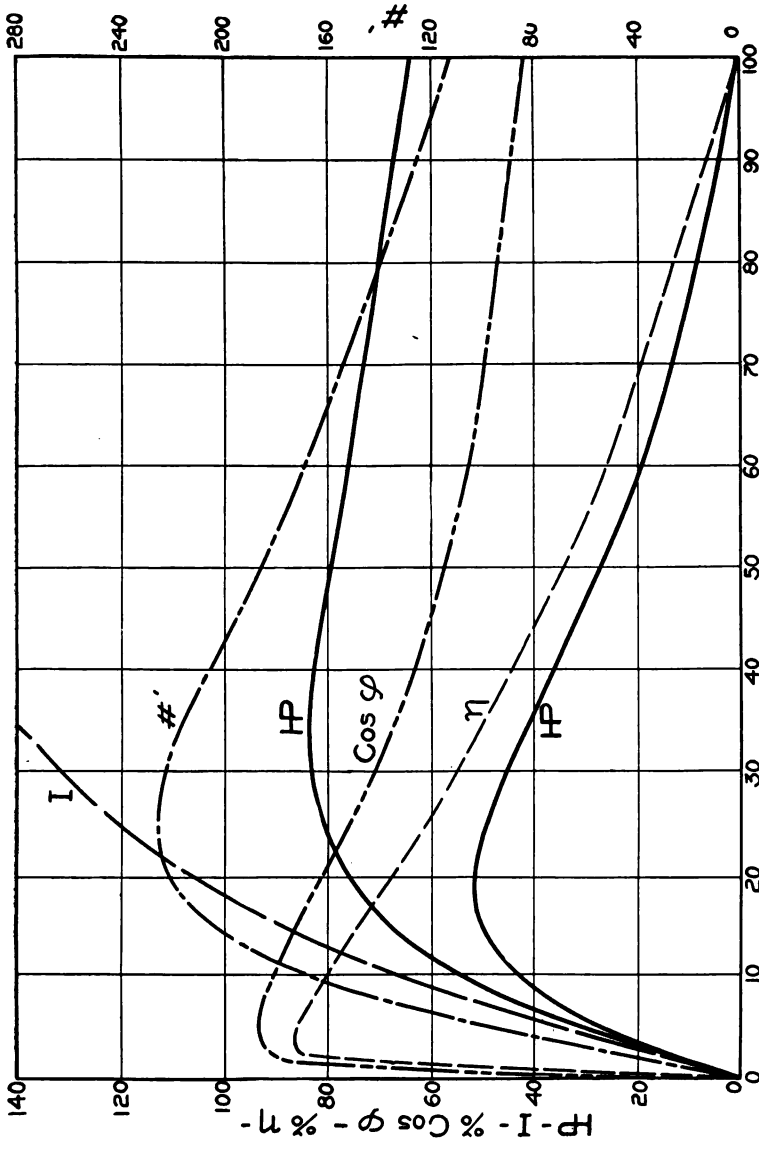



PLATE VIII.—OERLIKON 20 HORSE POWER 3 PHASE INDUCTION MOTOR.



Digitized by Google

power line $B_o'K'$. Erect perpendicular to the pressure line and lay off on it an arc 100 mm. long. Read directly in mm. or per cent., the efficiency at any load as the intersection of line O''B for that load with the efficiency line.

The motor was loaded both by brake and by a rated generator and the circle gave points between the fluctuations of the experimental data. The circle and curves derived therefrom follow.

OERLIKON MOTOR.
Data for circle, Grob.

| Amperes. | Slip. | Cos $\phi$ | $\eta$ | Torque lb. ft. | H. P. input. | H. P. output. | B. Q. | B. S. | B. G. |
|---|---|---|---|---|---|---|---|---|---|
| 10 | 0.9 | 68.8 | 70.9 | 18.2 | 6.8 | 4.7 | 71 | 53 | 60 |
| 20 | 2.2 | 90.9 | 84.7 | 48.9 | 16.2 | 13.3 | 183 | 150 | 161 |
| 30 | 3.9 | 93.4 | 86.5 | 79.0 | 25.1 | 21.5 | 284 | 243 | 260 |
| 40 | 5.4 | 93.6 | 85.6 | 104.8 | 33.4 | 28.7 | 378 | 325 | 345 |
| 50 | 6.9 | 93 | 84.0 | 130.6 | 41.6 | 35.3 | 472 | 400 | 430 |
| 75 | 11.4 | 89.5 | 78.3 | 182.4 | 59.2 | 46.4 | 670 | 525 | 600 |
| 100 | 17.3 | 83.4 | 70.2 | 217.0 | 74.0 | 52.4 | 837 | 593 | 714 |
| 125 | 25 8 | 74.5 | 59.1 | 226.0 | 82.5 | 48.9 | 934 | 554 | 744 |
| 150 | 42.5 | 61.4 | 41.8 | 200.5 | 81.8 | 34.2 | 926 | 387 | 660 |
| 174* | 100 | 41.5 | 0 | 112.5 | 64.0 | 0 | 735 | 0 | 320 |
| 7B† | 0.1† | 18.4 | 0 | 4 | 1.18 | 0 | 880‡ | 0 | ..... |
| 160 | 56 | 54.5 | 29.8 | 176 | 77.4 | 22.7 | 876 | 257 | 580 |

* Short circuit.
† No load.
‡ Watts.

GENERAL ELECTRIC CO.'S MOTOR No. 71602

Form L; 60 frequency; 5 h. p.; 3 phase; volts between lines, 110; 4 pole, amperes per line, 28. The rotor has an internal clutched starting resistance.

No load.

110 volts, 14.5 amperes, 420 watts, $\cos \phi_o = .1520$.

Short circuit.

| I (av.) | V | W (total). | Cos $\phi_k$ | $I_k$ (110 volts). | $W_k$ (110 volts). |
|---|---|---|---|---|---|
| 22.1 | 15.5 | 340 | .497 | 156 | 17160 |
| 26.3 | 18.5 | 480 | .493 | 156 | 17160 |
| 30.3 | 22.5 | 680 | .497 | 156 | 17160 |
| 35.0 | 26.0 | 890 | .484 | 156 | 17160 |
| 41.3 | 30.0 | 1,080 | .503 | 156 | 17160 |
| Av... .... | ..... | ..... ... | .494 | ..... | ..... ..... |

$$I_1^2 R_1^1 = I_2^2 R_2 = \frac{17160}{2} = 8580 = 156^2 \times .353$$



Digitized by Google

The windings of rotor and stator are identical, therefore $R_1' = R_2' = .353$.

Direct current measurements gave $R_1 = R_2 = .3115$ which is raised to .353 by eddy current losses.

$$C_0O^1 = 14.4,\ T_s = \frac{\frac{14.4}{\sqrt{3}} \times .353}{110} = .0267,\ T = \frac{OB_0}{B_0L} = .0870$$

Watt current of $I_2^2R_2$ loss is $\frac{8580}{3 \times 110} = 26$ amperes.

Circle, data scaled therefrom and curves follow.

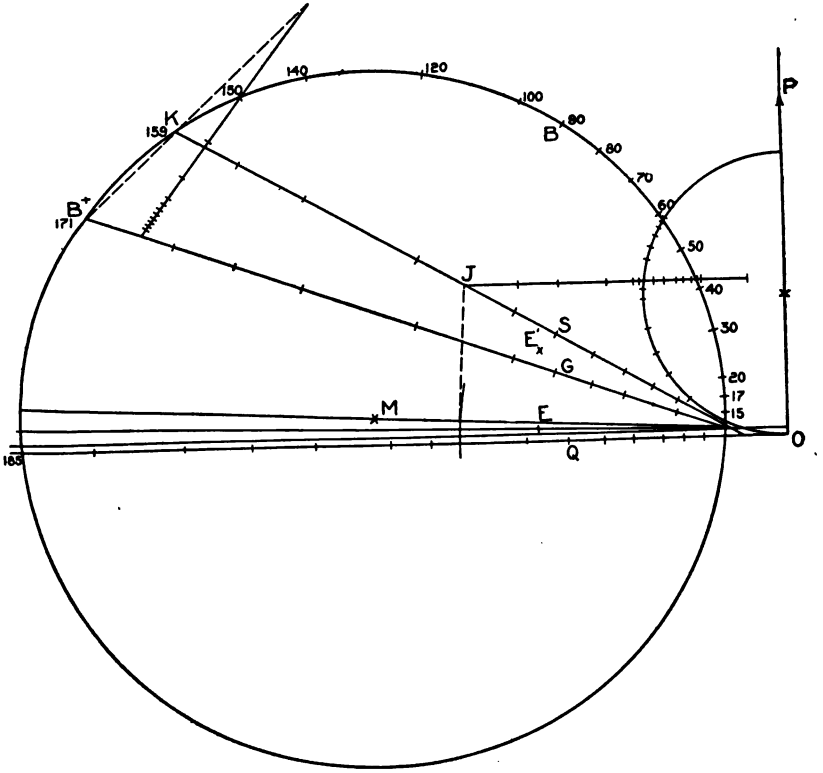



PLATE IX.—FIVE HORSE POWER INDUCTION MOTOR. G. E. No. 71602.



Digitized by Google

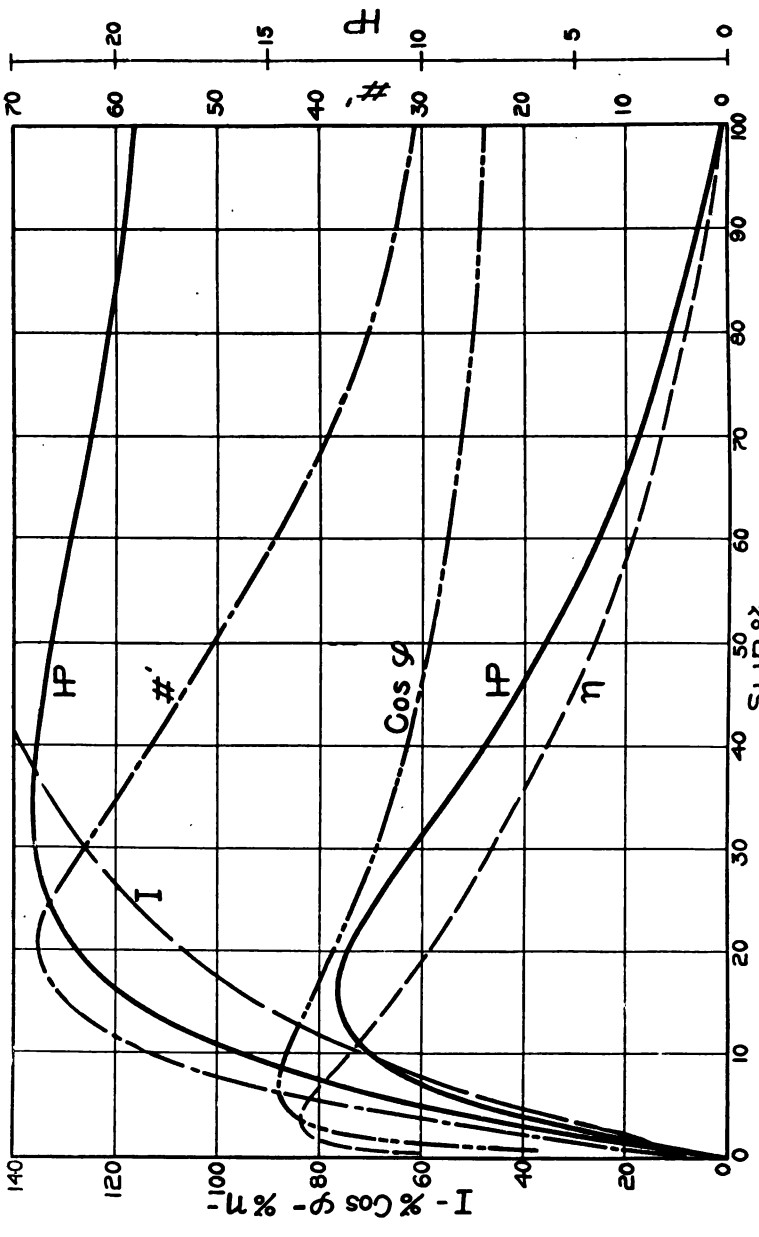



PLATE X.—FIVE HORSE POWER INDUCTION MOTOR. G. E. No. 71602.

[444]

Digitized by Google

GENERAL ELECTRIC MOTOR NO. 71602.

Data for circle: Kyser, Fussell.

| Amperes. | Slip. | Cos $\phi$ | Per ct. $\eta$ | Torque lb. ft. | H. P. Output | H. P. Input. | BQ | BS | BG |
|---|---|---|---|---|---|---|---|---|---|
| 15 | .50 | 37.5 | 60.2 | 4.78 | .85 | 1.53. | 90 | 50 | 65 |
| 17 | 1.15 | 55.5 | 74.8 | 8.81 | 1.78 | 2.46 | 145 | 105 | 120 |
| 20 | 1.90 | 64.5 | 80.2 | 13.4 | 2.81 | 3.73 | 210 | 165 | 182 |
| 30 | 3.5 | 83.8 | 83.6 | 25.0 | 5.35 | 6.46 | 380 | 315 | 340 |
| 40 | 4.9 | 86.3 | 82.6 | 34.6 | 7.33 | 8.83 | 520 | 432 | 470 |
| 50 | 6.2 | 88.0 | 80.3 | 43.0 | 9.08 | 11.28 | 665 | 535 | 585 |
| 60 | 7.9 | 87.4 | 77.6 | 50.0 | 10.36 | 13.40 | 790 | 610 | 680 |
| 70 | 10 1 | 86.3 | 74.5 | 56.6 | 11.52 | 15.40 | 910 | 680 | 770 |
| 80 | 11.9 | 84.5 | 70.8 | 61.0 | 12.20 | 17.30 | 1020 | 720 | 830 |
| 90 | 14.4 | 82.0 | 66.8 | 64.6 | 12.55 | 18.84 | 1110 | 740 | 880 |
| 100 | 17.3 | 79.3 | 62.2 | 66.9 | 12.42 | 20.30 | 1200 | 730 | 910 |
| 120 | 25.6 | 72.0 | 50.8 | 65.4 | 11.18 | 22.20 | 1310 | 660 | 890 |
| 140 | 42.0 | 62.0 | 33.3 | 55.2 | 7.36 | 22.20 | 1310 | 434 | 750 |
| 150 | 60.0 | 54.5 | 19 0 | 44.1 | 4.07 | 21.10 | 1240 | 240 | 600 |
| 159* | 100.0 | 47.0 | 00.0 | 30.1 | 0.00 | 19.00 | 1120 | 0 | 410 |

*Short circuit.

Values scaled from circle agree very well with the results taken from brake tests.

GENERAL ELECTRIC CO'S MOTOR NO. 2577

Type I; form C; 3 phase; 60 frequency; 6 pole; 5 horse power; 1200 rev. per min.; 220 volts. Stator turns, 1728; rotor turns, 144.

No load.

219 volts, $I_o = 3.88$ amperes, $\cos \phi_o = .176$.

Watts friction and windage $= 115$

Watt current friction and windage $= \frac{115}{220 \times \sqrt{3}} = .302$ amperes

Watt current total $= \frac{258}{220 \times \sqrt{3}} = .678$ amperes



Digitized by Google

Short circuit.

220 volts; 48 amperes; cos $\phi_k = .497$.

$R_1 = 1.21$ ohms (hot), $R_2 = .0086$ ohms (hot) $O'C_o = 3.64$.

$$T_s = \frac{\frac{3.64}{\sqrt{3}}}{220} \times 1.21 = .01125$$

$$T = \frac{OB_0}{B_0L} = .0716$$

$$48^2 \times 1.21 = 2790$$

$$\overline{12 \times 48}^2 \times .0086 = 2850$$

$$\frac{2850}{220} = 13 \text{ amperes watt current to subtract along line KK'.}$$

Circle, data and curves follow.

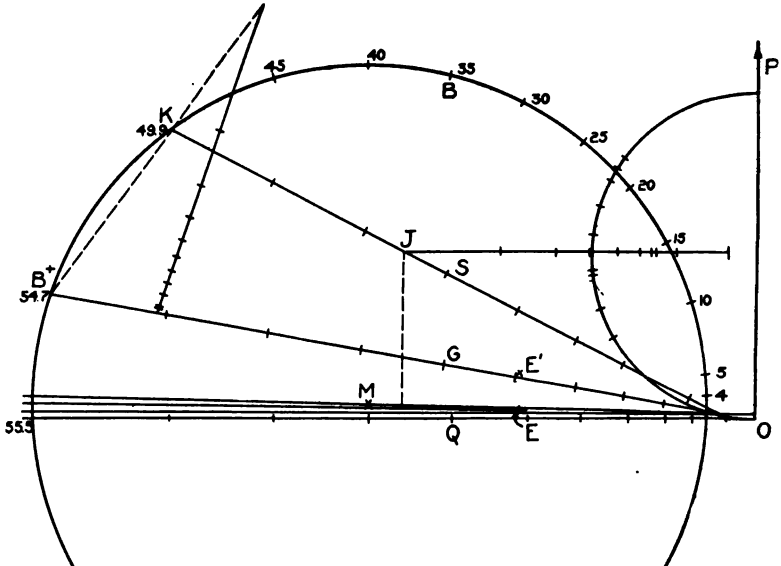



PLATE XI.—FIVE HORSE POWER INDUCTION MOTOR. G. E. No. 2577.

[446]

Digitized by Google

GENERAL ELECTRIC CO.'S MOTOR NO. 2577.

| Amperes. | Slip. | Cos $\phi$ | Per ct. $\eta$ | Torque lb. ft. | H. P. Input. | H. P. Output. | BQ | BS | BG |
|---|---|---|---|---|---|---|---|---|---|
| 3.88* | .4 | 17.6 | 0 | .6 | .346 | 0 | ..... | 0 | 11 |
| 4 | .9 | 41.0 | 54.4 | 2.8 | .854 | .459 | 67 | 36 | 50 |
| 5 | 1.9 | 66.2 | 76.0 | 6.5 | 1.78 | 1.29 | 140 | 101 | 118 |
| 10 | 5.8 | 87.6 | 84.0 | 18.1 | 4.52 | 3.75 | 355 | 294 | 327 |
| 15 | 9.4 | 89.2 | 81.4 | 27.2 | 6.89 | 5.61 | 540 | 440 | 492 |
| 20 | 13.2 | 87.4 | 77.2 | 35.7 | 9.04 | 7.00 | 708 | 548 | 645 |
| 25 | 18.0 | 84.8 | 72.4 | 42.0 | 10.84 | 7.84 | 850 | 614 | 760 |
| 30 | 23.4 | 80.2 | 65.9 | 46.6 | 12.33 | 8.15 | 967 | 639 | 844 |
| 35 | 31.0 | 74.5 | 57.8 | 49.3 | 13.40 | 7.78 | 1050 | 610 | 890 |
| 40 | 41.5 | 67.2 | 46.9 | 48.4 | 13.80 | 6.50 | 1080 | 510 | 875 |
| 45 | 59.0 | 57.0 | 30.0 | 43.2 | 13.27 | 4.08 | 1040 | 320 | 750 |
| 48 | 79.0 | 49.7 | 14.4 | 36.7 | 12.20 | 1.79 | 956 | 140 | 664 |
| 49.9† | 100.0 | 44.2 | 0 | 31.2 | 11.22 | 0 | 880 | 0 | 565 |

*No load.
†Short circuit.

## WESTERAS MOTOR

(Allmanna Svenska Elektriska Aktiebolaget.)

From H. M. Hobart's book *Electric Motors.*

12 pole; 50 frequency; 500 volt; 500 rev. min.; 3 $\phi$; 100 horse power, Y connected, 288V per coil.

$$\text{Per } \phi \begin{cases} R_1 = .084 \qquad R_2 = .058 \\ \dfrac{\text{Primary turns}}{\text{Secondary turns}} = \dfrac{90}{72} = 1.25 \end{cases}$$

No load.

23 amperes, $\cos \phi_0 = .190$

$3 \times 288 \times 23 \times .19 = 3700$ watts

Bearing and windage = 1400 watts

$$\text{Watt current total} = \frac{3700}{500 \times \sqrt{3}} = 4.27 \text{ amperes}$$

$$\text{Watt current, bearing and windage} = \frac{1400}{500 \times \sqrt{3}} = 1.61 \text{ amperes.}$$



Digitized by Google

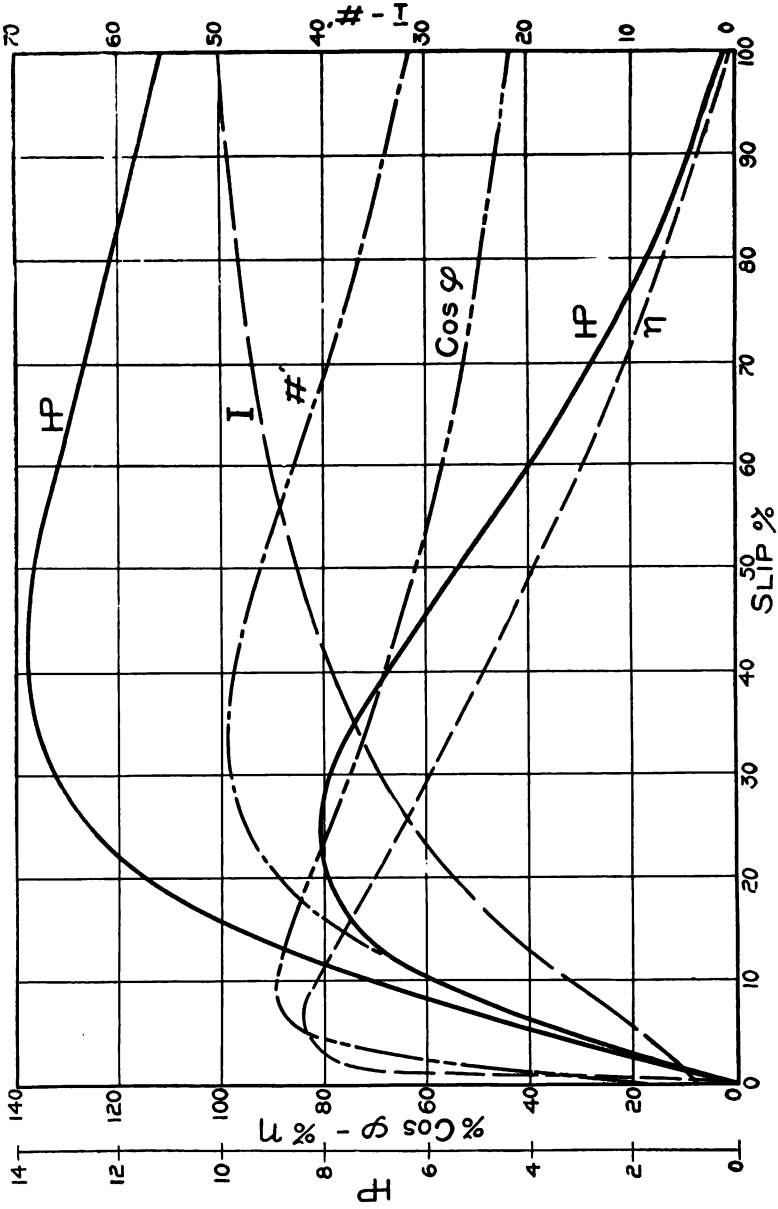



PLATE XII. FIVE HORSE POWER INDUCTION MOTOR. G. E. No. 2577.

[448]

Digitized by Google

Short circuit.

438 amperes; 288 volts; cos $\phi_k = .383$.

$O^1C_0 = 22.4$ amperes

$22.4 \times .084 = 1.88$ volt

$$T_3 = \frac{1.88}{\left(\frac{500}{\sqrt{3}}\right)} = .00651$$

$$T = \frac{OB_0}{B_0L} = \frac{1.12}{22.51} = .0498$$

$I_2^2R_2 = (438 \times 1.25)^2 \cdot 058 \times .383 = 65000$ watts

$$\text{Watt current at short} = \frac{65000}{500 \times \sqrt{3}} = 75.2 \text{ amperes}$$

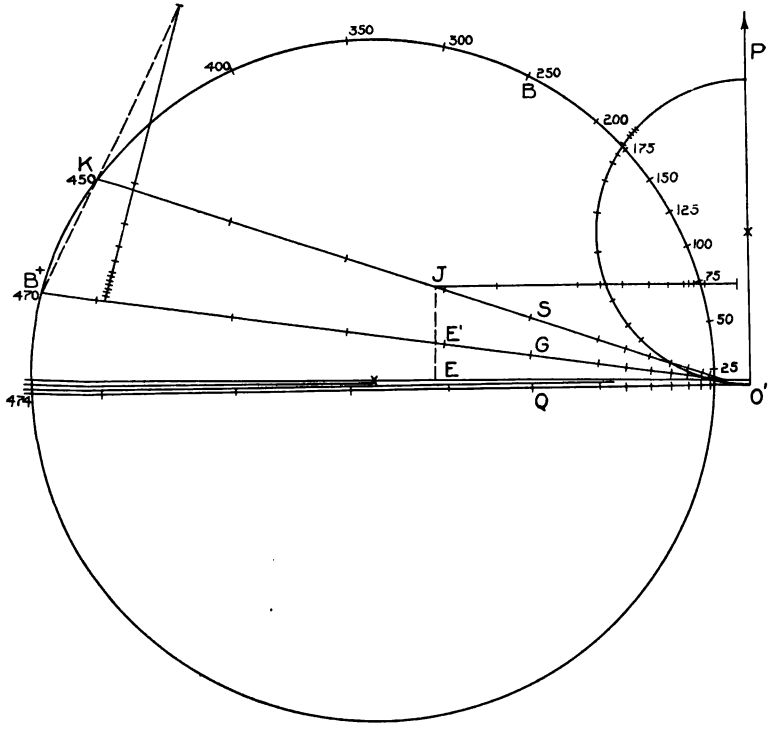



PLATE XIII.—WESTERAS 100 HORSE POWER 3 PHASE INDUCTION MOTOR.



Digitized by Google

WESTERAS 100 HORSE-POWER MOTOR.

| Ampere. | Slip. | Cos $\phi$ | $\eta$ | Torque lb. ft. | H. P. Input. | H. P. Output. | BG. | BQ. | BS. |
|---|---|---|---|---|---|---|---|---|---|
| 23 | .05 | 190 | 0 | 21.6 | 5 | 0 | 9 | 37 | 0 |
| 25 | .2 | 440 | 547 | 98.4 | 13.5 | 7.9 | 41 | 58 | 34 |
| 50 | 1.8 | 853 | 860 | 460 | 48.7 | 41.7 | 192 | 210 | 180 |
| 75 | 3.0 | 905 | 880 | 768 | 80.6 | 69.6 | 320 | 335 | 300 |
| 100 | 4.1 | 917 | 881 | 1030 | 106.8 | 93.4 | 430 | 460 | 402 |
| 125 | 5.2 | 912 | 870 | 1292 | 133 | 116.0 | 538 | 573 | 500 |
| 150 | 6.8 | 902 | 858 | 1519 | 158 | 136.0 | 632 | 680 | 581 |
| 175 | 8.0 | 889 | 841 | 1730 | 181 | 152.5 | 721 | 780 | 657 |
| 200 | 9.7 | 870 | 822 | 1920 | 202 | 166.5 | 800 | 876 | 718 |
| 250 | 13.5 | 817 | 776 | 2210 | 238 | 184.5 | 922 | 1028 | 796 |
| 300 | 17.4 | 750 | 713 | 2360 | 262 | 186.5 | 983 | 1130 | 804 |
| 350 | 25.3 | 660 | 628 | 2305 | 268 | 167.0 | 960 | 1155 | 720 |
| 400 | 39.4 | 532 | 474 | 1960 | 246 | 116.0 | 815 | 1061 | 500 |
| 438 | 71.0 | 383 | 207 | 1320 | 195 | 40.4 | 550 | 840 | 174 |
| 450 | 100 | 320 | 0 | 980 | 164 | 0 | 400 | 708 | 0 |

## BROWN, BOVERI MOTOR NO. 193,913

Type 6; 6 horse power; 4 pole; 1800 rev. per min.; 3 phase; 110 volts; 60 frequency.

$R_1' = .137$ ohms (hot)

$R_2' = .089$ ohms (hot)

No load.

110 volts, $I_o = 8.45$ amperes, $\cos \phi_o = .290$

160 Watts friction and windage

Watt current friction and windage $\frac{160}{110 \times \sqrt{3}} = .84$ amperes

Watt current total $\frac{466}{110 \times \sqrt{3}} = 2.45$ amperes

Short circuit.

110 volts, $I_k = 159$ amperes, $\cos \phi_k = .572$

$$T_s = \frac{8. \times .137}{110} = .00996$$

$$T = .0432$$

Plate XVI shows this machine as a normal motor, solid line, and the dotted line shows a modified motor which is discussed later. Circle, data and curves are given. Plate XVIIa shows the curves for this machine and Plate XVIIb for the modified motor.



Digitized by Google

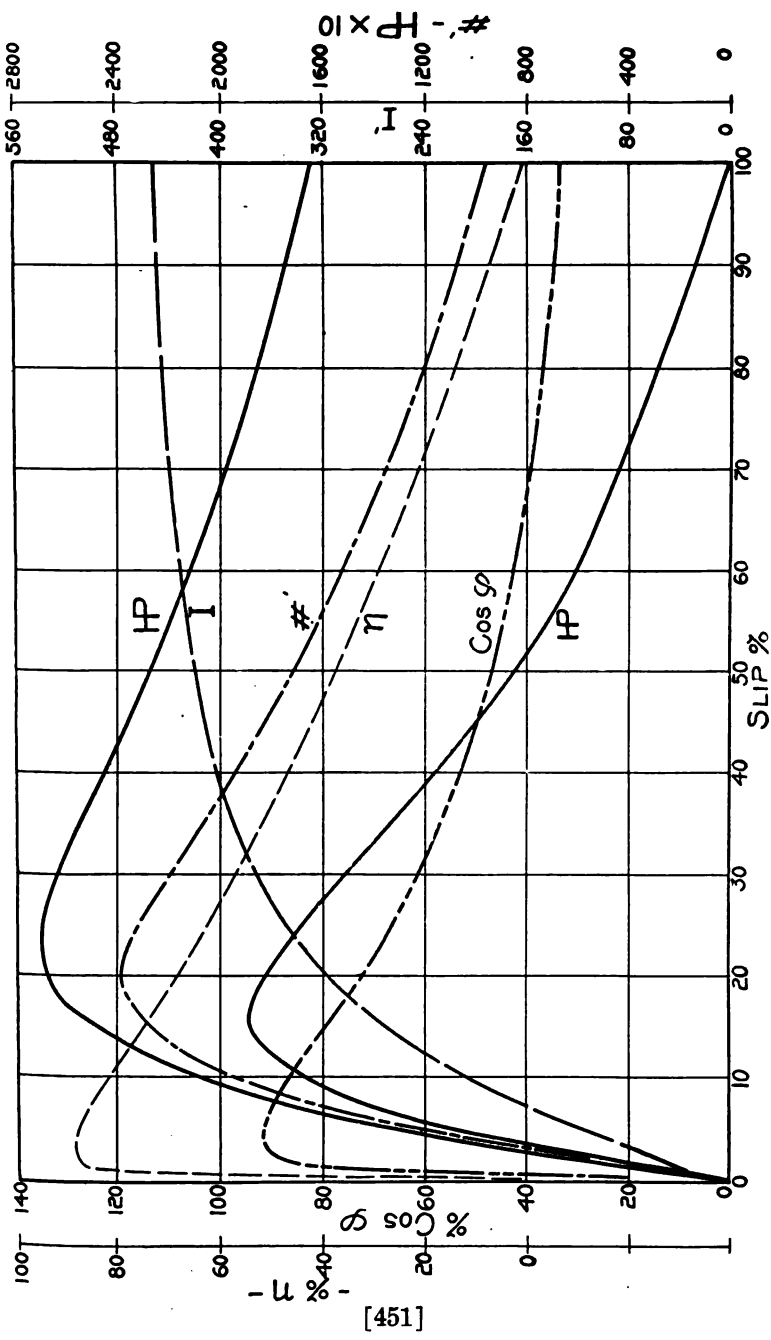



PLATE XIV.—WESTERAS 100 HORSE POWER INDUCTION MOTOR.

[451]

Digitized by Google

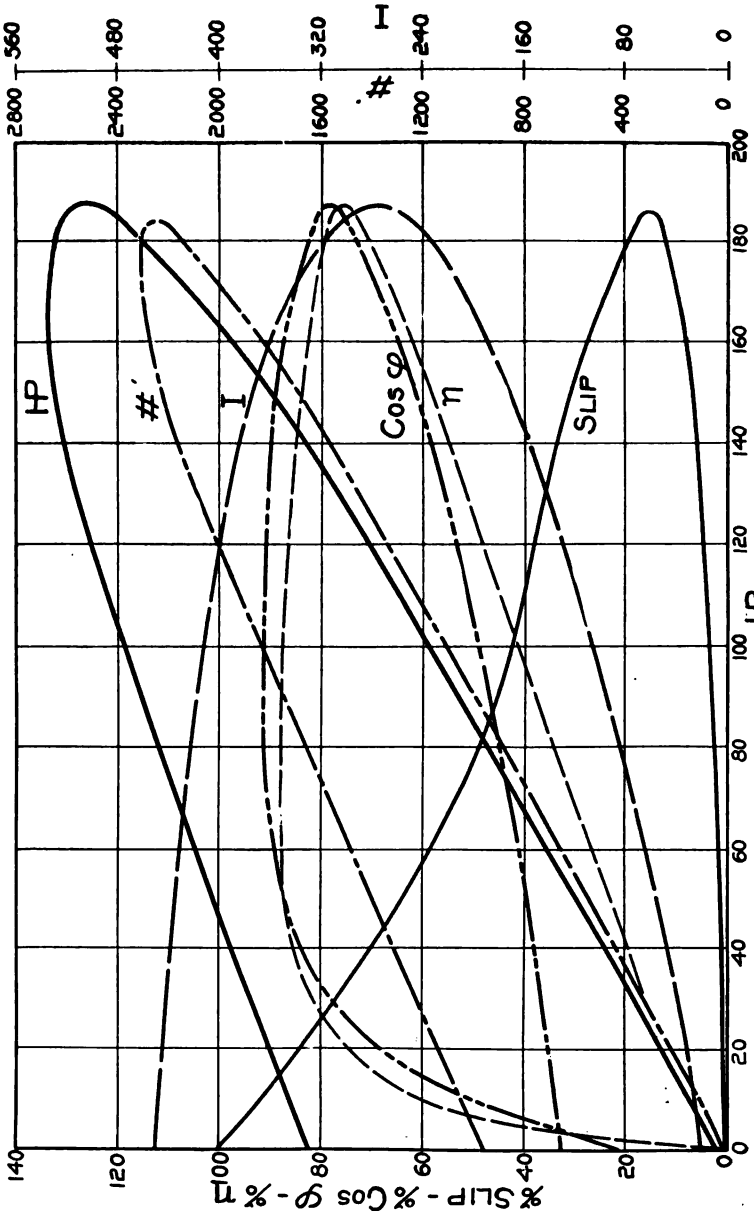



PLATE XV.—WESTERAS 3 PHASE INDUCTION MOTOR.



Digitized by Google

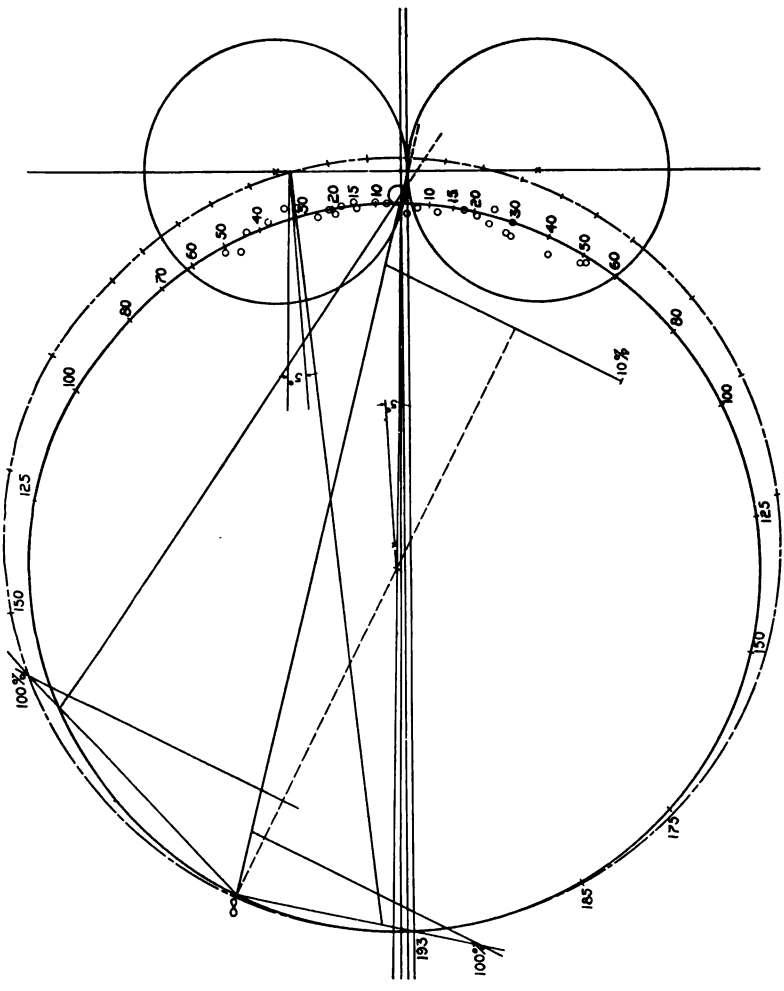



PLATE XVI.—GROB. DIAGRAM OF BROWN BOVERI 3 PHASE INDUCTION MOTOR.
(Normal and compensated with 5° brush displacement.) Small circles indicate experimental values.



Digitized by Google

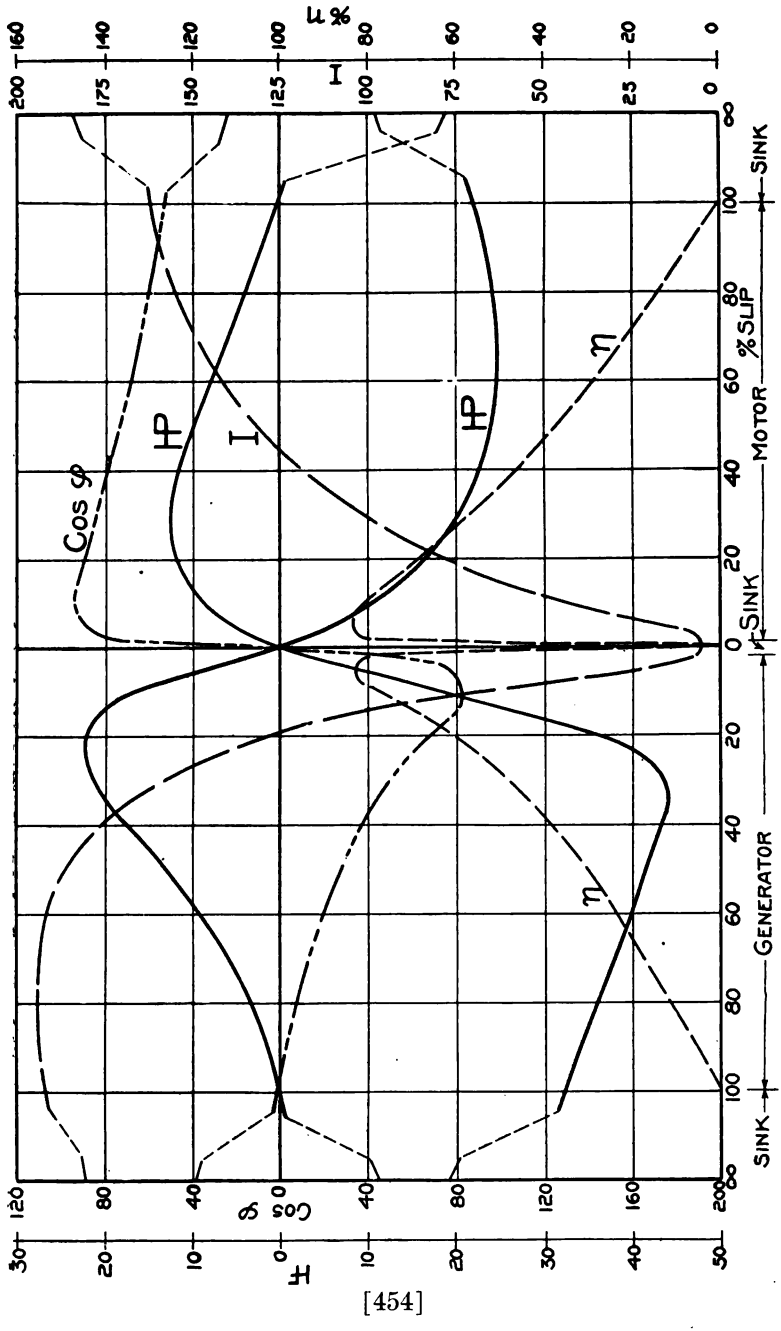



PLATE XVIIa.—BROWN BOVERI 5 HORSE POWER 3 PHASE INDUCTION MOTOR.
Normal operation.



Digitized by Google

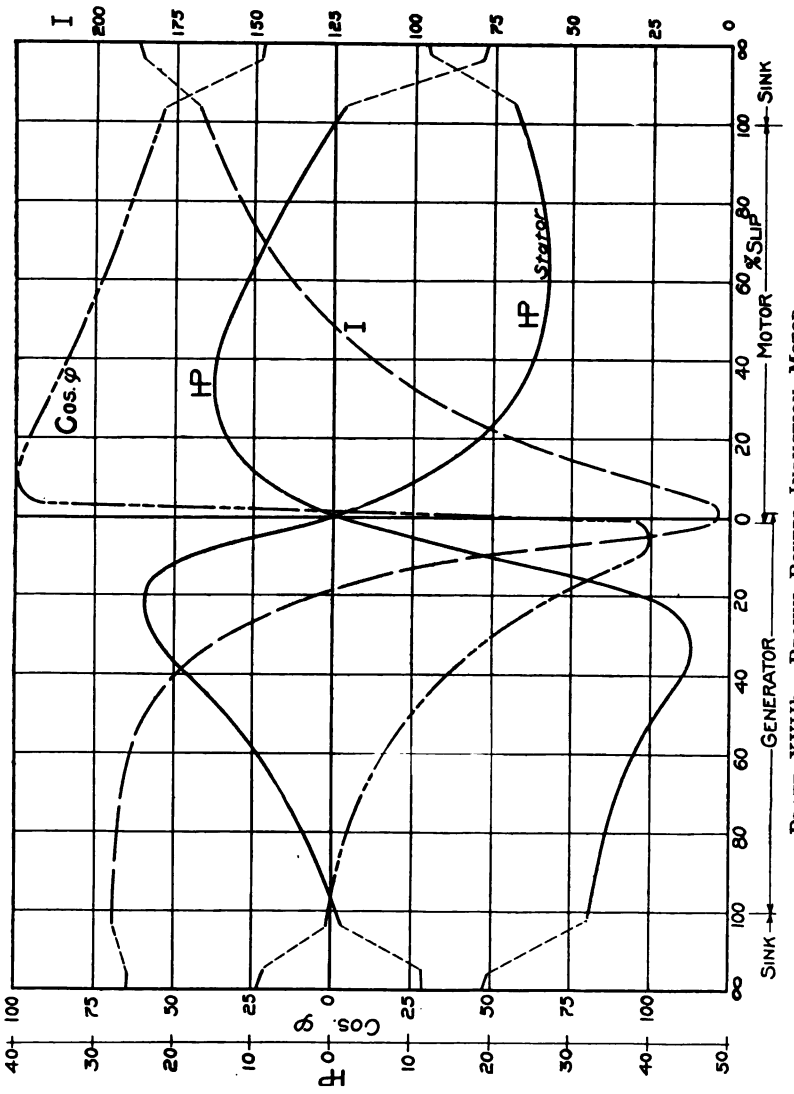



PLATE XVIIb.—BROWN BOVERI INDUCTION MOTOR.
Compensated 5° Brush displacement.



Digitized by Google

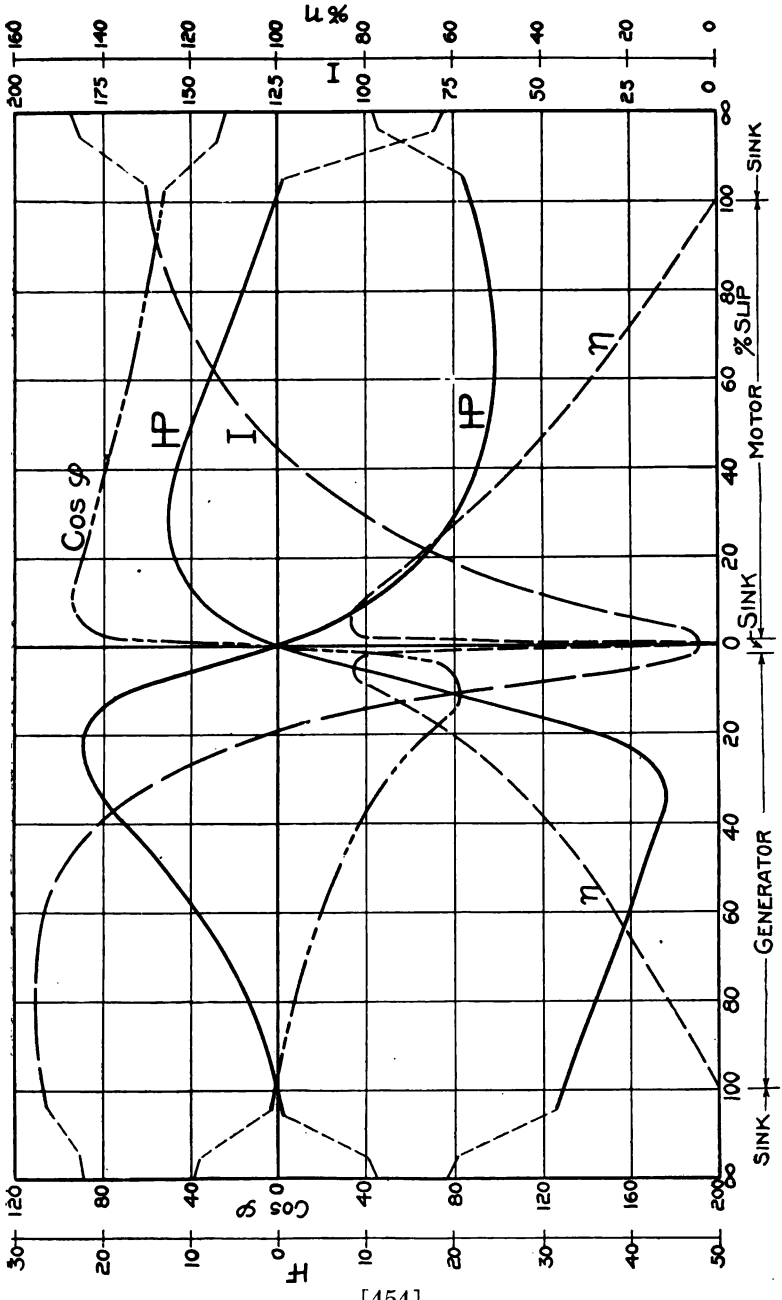



PLATE XVIIa.—BROWN BOVERI 5 HORSE POWER 3 PHASE INDUCTION MOTOR.
Normal operation.



Digitized by Google

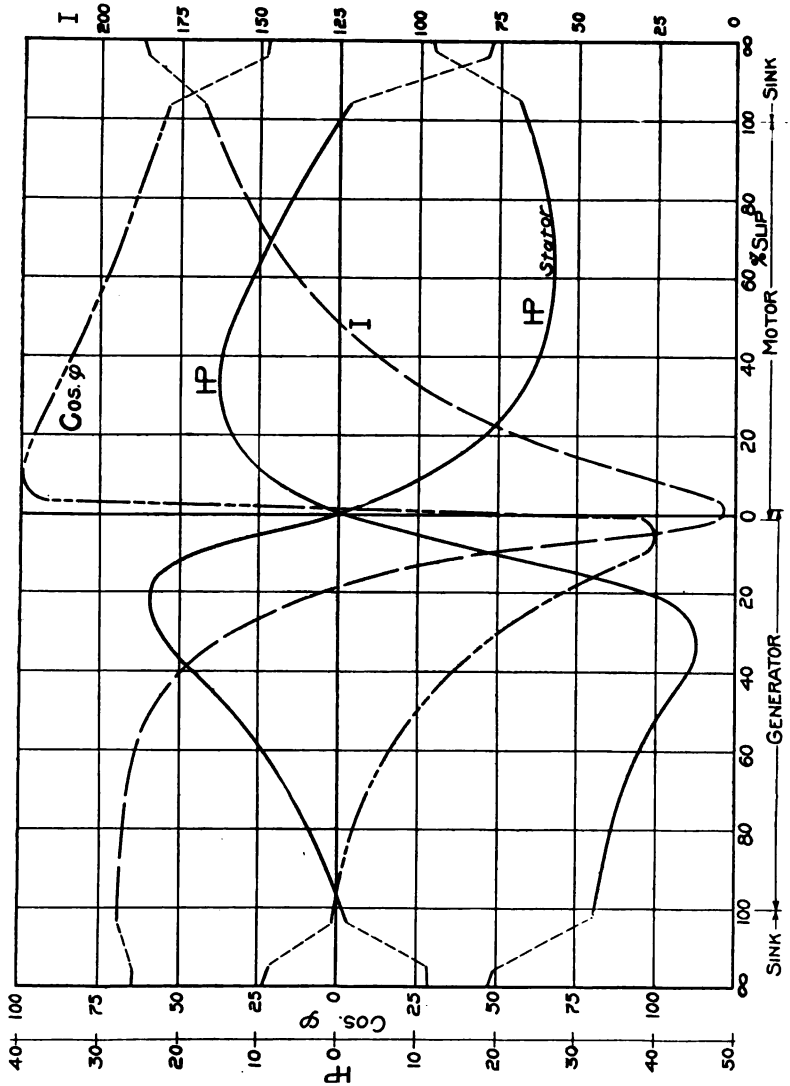



PLATE XVIIb.—BROWN BOVERI INDUCTION MOTOR.
Compensated 5° Brush displacement.



Digitized by Google

| Per cent. slip. | H. P. mechanical. | H. P. electrical. | $I_1$ | Cos $\phi$ | $\eta$ |
|---|---|---|---|---|---|
| ∞ Standstill | —19.80 | 11.80 | 188 | 24.0 | ..... |
| 1.00 | 0.00 | 23.20 | 162 | 55.0 | 0.00 |
| .717 | 4.68 | 24.60 | 150 | 62.5 | 19.0 |
| .446 | 10.40 | 24.40 | 125 | 75.0 | 42.5 |
| .218 | 12.60 | 21.82 | 100 | 83.5 | 58.0 |
| .212 | 12.30 | 18.36 | 80 | 88.0 | 67.0 |
| .180 | 11.60 | 16.30 | 70 | 90.6 | 71.3 |
| .148 | 10.57 | 14.20 | 60 | 91.8 | 74.4 |
| .119 | 9.35 | 12.10 | 50 | 92.9 | 77.1 |
| .094 | 7.80 | 9.70 | 40 | 93.5 | 80.4 |
| .068 | 5.88 | 7.28 | 30 | 92.7 | 81.0 |
| .040 | 3.81 | 4.68 | 20 | 89.0 | 81.5 |
| .030 | 2.43 | 3.12 | 15 | 82.0 | 77.7 |
| .010 | 1.04 | 1.73 | 10 | 61.0 | 60.0 |
| .0098 | 0.52 | 1.21 | 9 | 45.0 | 42.9 |
| Synchronism | | | | | |
| .0000 | 0.00 | .69 | 8.06 | 29.0 | 0.00 |
| .0097 | 2.25 | 1.38 | 10 | 54.0 | 61.5 |
| .0176 | 3.81 | 3.12 | 15 | 78.2 | 81.7 |
| .0247 | 5.36 | 4.33 | 20 | 86.0 | 80.6 |
| .0382 | 8.50 | 6.93 | 30 | 90.3 | 81.6 |
| .0519 | 11.60 | 9.35 | 40 | 91.0 | 80.6 |
| .0646 | 14.50 | 11.60 | 50 | 90.5 | 79.7 |
| .0780 | 18.20 | 13.70 | 60 | 89.0 | 75.2 |
| 0.1042 | 23.9 | 17.5 | 80 | 85.4 | 73.2 |
| 0.1370 | 30.3 | 20.8 | 100 | 80.0 | 68.6 |
| 0.1800 | 37.3 | 23.0 | 125 | 71.0 | 62.0 |
| 0.2500 | 43.2 | 22.5 | 150 | 58.0 | 52.5 |
| 0.3800 | 44.4 | 17.0 | 175 | 37.0 | 38.3 |
| 1.0000 | 33.0 | 0. | 192 | 0 | 0. |

BROWN, BOVERI, MOTOR BRAKE TEST.

| I av. | Per cent. Cos $\phi$ | Slip. | B. H. P. |
|---|---|---|---|
| 8.25 | 25.0 | .22 | .0 |
| 11.3 | 71.8 | 1.50 | 1.35 |
| 15.9 | 82.1 | 3.0 | 2.68 |
| 18.8 | 88.5 | 3.5 | 3.34 |
| 21.9 | 90.1 | 4.5 | 3.98 |
| 25.2 | 88.8 | 5.2 | 4.63 |
| 29.0 | 95.1 | 6.5 | 5.26 |
| 32.6 | 96.0 | 7.3 | 5.90 |
| 37.4 | 94.2 | 9.1 | 7.15 |
| 43.3 | 93.5 | 9.45 | 7.79 |
| 47.0 | 90.5 | 10.5 | 8.38 |
| 50.5 | 91.6 | 11.0 | 8.68 |
| 8.2 | 26.3 | .187 | 0 |
| 9.8 | 49.2 | .86 | 1.14 |
| 15.3 | 85.2 | 1.42 | 2.46 |
| 20.7 | 86.7 | 2.28 | 3.72 |
| 32.6 | 87.9 | 4.02 | 5.48 |



Digitized by Google

## Slip Scale for Hypersynchronism

In all of the foregoing diagrams, as well as in those given in the appendix to this chapter, the scales for slip are accurate for the condition of motor running since they represent all the limit conditions and agree with brake data. For generator slip, however, none are accurate since the authors have done one of three things: (a) evaded the issue entirely, Grob, 1904; (b) extended the motor slip scale proportionally which fails to represent the limit conditions, Ossanna; or (c) extended the generator slip scale set by the limits to motor action which fails to give the correct position for 100 per cent. motor slip (standstill), Grob, 1901.

If in a diagram, whose motor slip scale accurately agrees with data which are obtained from careful observation during a brake test, the motor slip line be prolonged the values scaled off from this prolongation do not agree with tests made during a generator run. In one test at about 50 per cent. overload they had the relation, scale to test equals 10.7 to 6.1, not a remarkable agreement.

If, however, parallel to the motor slip scale a line be drawn, so that its 100 mm. length fills the distance between the base line of slip and a line drawn from $\infty$ slip through Z the point where the energy component of the primary current changes direction and which must represent the point of 100 per cent. slip, it will represent to a very fair degree of approximation the conditions actually found to occur during careful measurements of a run of the machine as a generator.

The diagrams given show two of the possible schemes of connection, each employing a synchronous generator in parallel as the source of leading exciting current. One scheme loads with lamps or other dead resistance as absorption agent, while the other is a "pump back" system. The latter is the easier to operate by far; it being possible by the mere movement of a rheostat (that in the smaller DC machine "R" being most often used) to effect a change of the induction machine clear through by easy steps from 100 per cent. overload as a motor to 100 per cent. overload as a generator. This causes readily 20 h. p. to



Digitized by Google

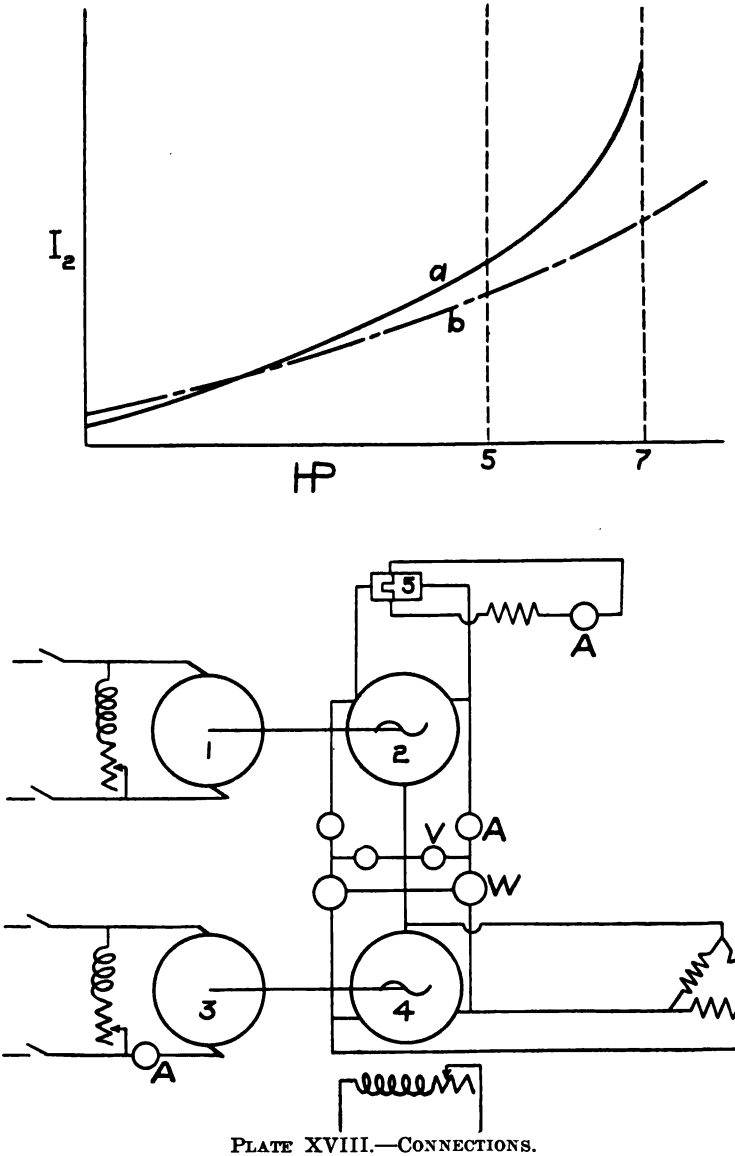



PLATE XVIII.—CONNECTIONS.



Digitized by Google

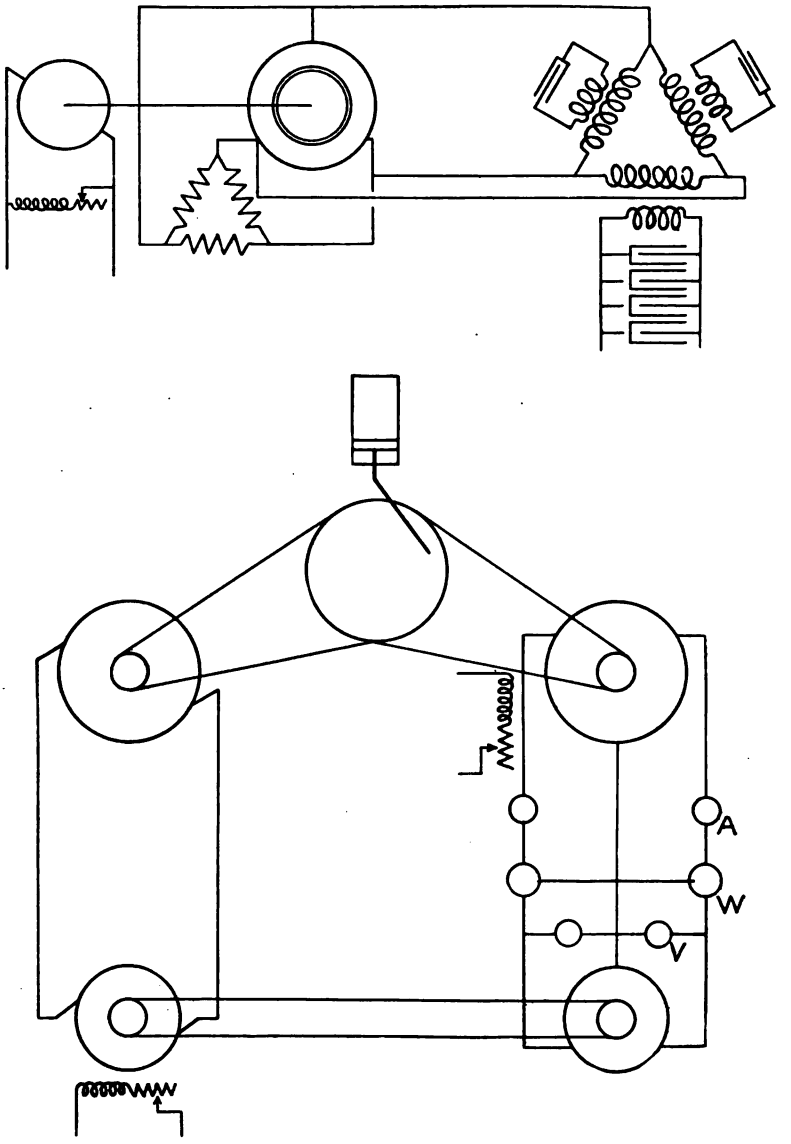



PLATE XIX.—CONDENSER EXCITATION AND PUMP. BACK CONNECTIONS.



Digitized by Google

circulate with a total loss of less than 2 h. p., the machine under test being rated at 6 P. S. (German h. p. of 736 watts).

In order to test this theory, the following experiment was performed. Driving above speed an induction machine it was paralleled with a synchronous generator and the following values maintained constant while load was added in the shape of lamp banks in delta across the lines, primary frequency, held at 60 by keeping the speed of the synchronous generator at 1,200 rev. per min.; the line pressure by adjusting the synchronous machine field current; and approximately the input to the machine driving the synchronous generator and therefore to the synchronous generator itself. It was found that while remarkably stable and constant when once adjusted, the system was one of a very considerable flexibility, since it sufficed to change any one of the three field rheostats to throw the load of the whole system on to whichever prime mover was desired and to cause the other to act as a generator, or both could be adjusted to act as motors. The motor driving the synchronous generator, in order that the frequency might be maintained constant, was kept functioning as a motor but was not allowed to take more than a very little power input. Accurate readings of speed were taken on both generators and these were used as checks on the indications of the slip-indicator. This consists (See Plate XVIII) of a commutator capable of being driven by the rotor speed and having a number of segments equal to the number of poles. Stator frequency impressed electrically upon these bars and rotor frequency mechanically (by holding against the shaft as in a tachometer) the number of beats due to these two frequencies superimposed is indicated by the throws of a DC ammeter needle connected in the circuit of the indicator brushes. For a four-pole motor the indications per half minute equal the revolutions slip per minute.

For the low readings if the tachometers and slip indicator disagreed the latter was taken, when the slip rose to higher values (due to the difficulty of counting) discrepancies were adjusted by weighting the slip-indicator as four times as accurate as the tachometer. All the values are the average of a



Digitized by Google

number of separate determinations taken after the constants had been adjusted and the performance stationary. The limit was placed by the sparking of the Westinghouse motor and not by the induction generator although it did get warm.

The slip $\overline{\lfloor\!\rfloor}$ is that scaled from the prolongation of the motor slip line while the slip S is from test values. The columns headed slip from XY and from YZ indicate the agreement obtainable from two diagrams, for the same motor, constructed entirely independently of one another, when scaled off my line.

The curves, Plates XX and XXI, between per cent. slip from test and per cent. slip from scale on my diagram is a most drastic test under which the diagram stands up well. The larger circles as indicated, represent the values as scaled from the correct lines, while the smaller circles are the values of generator slip as scaled from the prolongation of the motor slip line.

The agreement of the motor values with the 45° line is well-nigh perfect and the great improvement of the generator slip line is apparent to the most casual inspection. The greatest error of my values is large, 46.6 per cent., the average being high, however, about 15 per cent. or less.

The values of the prolonged motor slip line, however, are very much greater, the maximum *error* being 132 per cent. of the true value, while the average is more than 100 per cent. This means that the generator slip in diagrams heretofore employed gave values which were twice the real values as obtained by test.



Digitized by Google

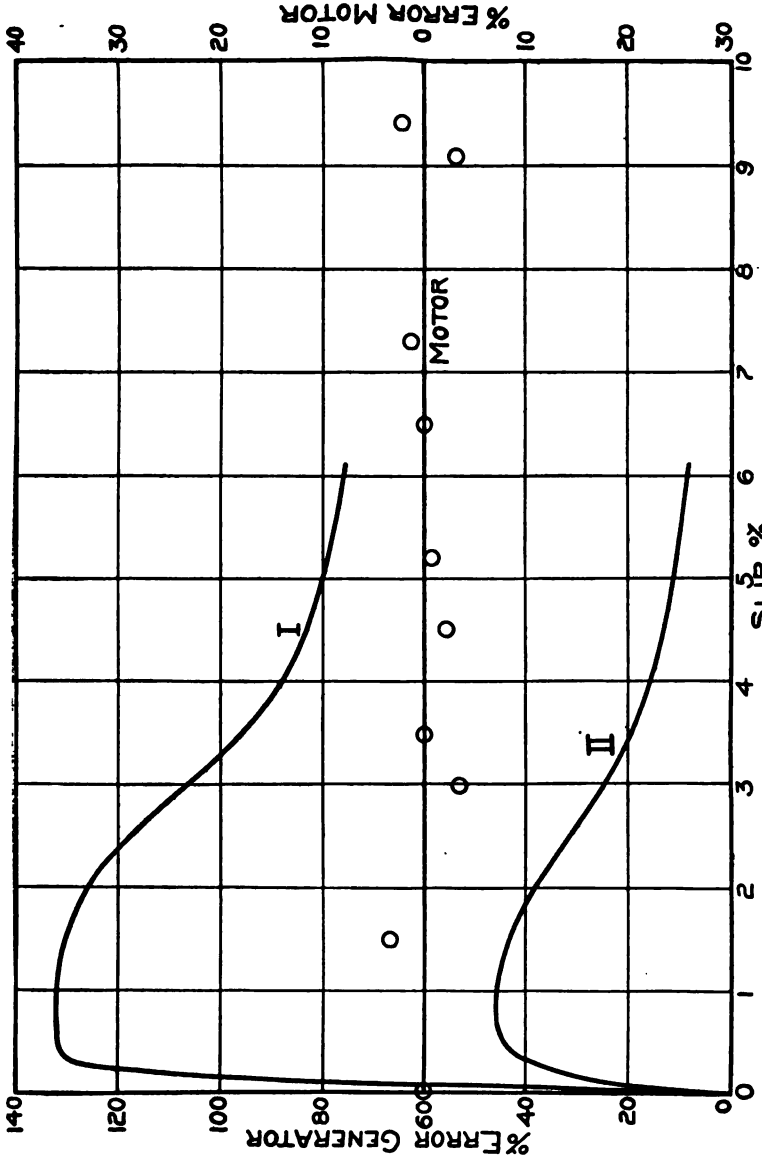



PLATE XX.—CURVE I FROM MOTOR SLIP LINE CONTINUED. CURVE II FROM CORRECTED SLIP LINE.



Digitized by Google

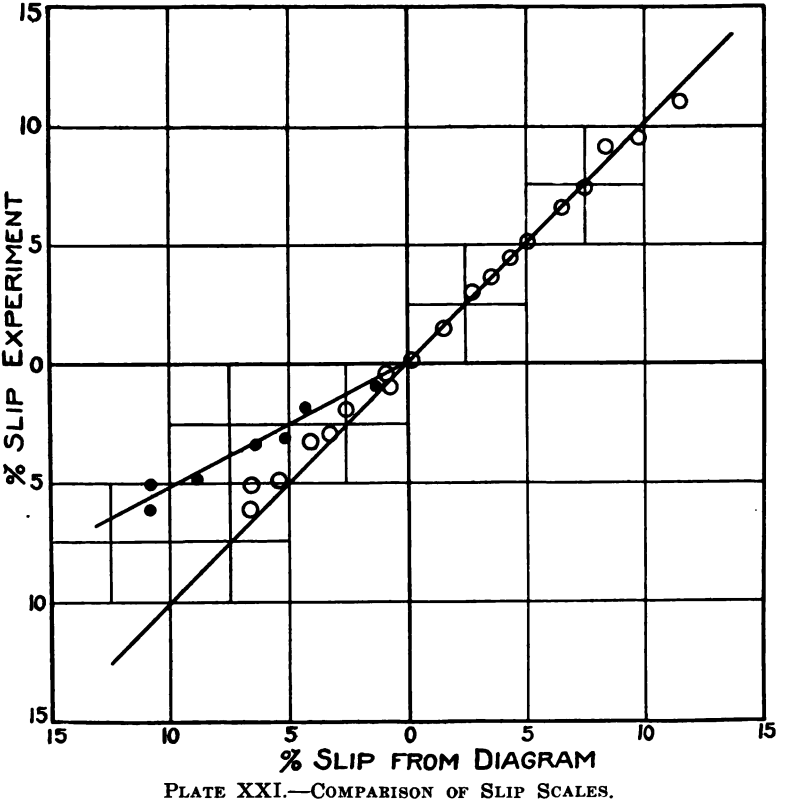



PLATE XXI.—COMPARISON OF SLIP SCALES.
Small circles give values from continuing the motor slip line.



Digitized by Google

| No. | $A_1$ av. | $V_1$ av. | $KW_1$ | $KW_2$ | KW total. | $IE1'\bar{3}$ |
|---|---|---|---|---|---|---|
| 1 | 9.8 | 110.6 | .91 | 1.01 | 1.92 | 1.870 |
| 2 | 21.3 | 109.2 | 2.25 | 1.14 | 3.39 | 4.040 |
| 3 | 31.3 | 109.3 | 3.35 | 1.67 | 5.02 | 5.940 |
| 4 | 41.9 | 108.5 | 4.42 | 2.33 | 6.75 | 7.860 |
| 5 | 50.8 | 110.2 | 5.57 | 2.91 | 8.48 | 9.680 |

| | Cos $\phi$ | Stator rev. per min. | Rotor rev. per min. | Slip by difference. | Slip by indicator. | Slip rev. per min. |
|---|---|---|---|---|---|---|
| 1 | 1.035 | 1,206.5 | 1.821 | 11 | 7.5 | 7.5 |
| 2 | .840 | 1,206.5 | 1,845 | 35 | 34 | 34 |
| 3 | .845 | 1,203 | 1,868 | 60 | 58 | 58.5 |
| 4 | .858 | 1,200 | 1,889 | 89 | 85 | 86.8 |
| 5 | .888 | 1,200 | 1,904 | 104 | 112 | 110 |

| | Slip S per cent. | Slip $\mathfrak{S}$ per cent. | Line frequency. | Lamps. | Westinghouse. | |
|---|---|---|---|---|---|---|
| | | | | | A total. | V |
| 1 | .41 | .95 | 60.3 | 0 | 16.8 | 110.8 |
| 2 | 1.88 | 4.30 | 60.3 | 3 x 10 | 43.6 | 109.9 |
| 3 | 3.24 | 6.3 | 60.2 | 3 x 17 | 67.6 | 109.2 |
| 4 | 4.83 | 8.8 | 60.0 | 3 x 24 | 98.0 | 108.7 |
| 5 | 6.11 | 10.7 | 60.0 | 3 x 32 | 131.0 | 105.0 |

| | Western A armature. | G.E.I. f | H. P. Output. | Slip from diagram. | |
|---|---|---|---|---|---|
| | | | | Xy | yZ |
| 1 | 16.0 | 4.68 | ..... | .6 | .60 |
| 2 | 16.6 | 4.62 | 4.54 | 2.6 | 2.63 |
| 3 | 16.2 | 4.75 | 6.72 | 4.1 | 3.94 |
| 4 | 17.0 | 4.87 | 9.04 | 5.4 | 5.38 |
| 5 | 17.2 | 4.90 | 11.39 | 6.6 | 6.59 |



Digitized by Google

| No. | $A_1$ | $A_2$ | A av. | $KW_1$ | $KW_2$ | KW total. |
|---|---|---|---|---|---|---|
| 1 | 9.4 | 10.9 | 10.45 | .855 | —.224 | .631 |
| 2 | 25.6 | 23.8 | 25.10 | 1.465 | 2.620 | 4.085 |
| 3 | 30.8 | 29.8 | 30.70 | 1.865 | 3.310 | 5.175 |
| 4 | 50.8 | 50.0 | 50.40 | 3.070 | 5.675 | 8.745 |

| No. | Cos $\phi$ | $V_1N$ | $V_2N$ | V av. | Rev. per min. | |
|---|---|---|---|---|---|---|
| | | | | | Rotor. | Stator. |
| 1 | .318 | 110 | 111 | 110.5 | 1,810 | 1,200 |
| 2 | .841 | 112 | 111 | 111.5 | 1,853 | 1,200 |
| 3 | .853 | 113 | 112 | 112.5 | 1,844 | 1,195 |
| 4 | .885 | 113 | 113 | 113.0 | 1,902 | 1,200 |

| No. | Slip by | | Slip rev. per min. | Slip S per cent. | $\frac{S}{5}$ per cent. | Cycles. |
|---|---|---|---|---|---|---|
| | Difference. | Indicator. | | | | |
| 1 | 10 | 17 | 17 | .945 | 1.0 | 60 |
| 2 | 53 | 53 | 53 | 2.94 | 5.1 | 60 |
| 3 | 52 | 61 | 59 | 3.30 | 6.4 | 59.7 |
| 4 | 102 | 88 | 91 | 5.05 | 10.7 | 60 |

| No. | Lamps. | Amperes to rotary. | H. P. output. | X Y | Y Z | |
|---|---|---|---|---|---|---|
| 1 | 0 | 14 | .846 | .70 | .63 | |
| 2 | 3x10 | 4 | 5.47 | 3.2 | 3.14 | |
| 3 | 3x15 | 4 | 6.94 | 4.0 | 3.82 | |
| 4 | 3x25 | 1.5 | 11.7 | 6 5 | 6.47 | |



Digitized by Google

## APPENDIX TO CHAPTER I

In chapter I credit must be given Adolph Thomälen for his work of November 26, 1903, and August 11, 1904, in *Elecktrotechnische Zeitschrift* following along the lines of Grob's diagram and giving the relation of the Heyland to the Ossanna circle. Of course, Heyland in 1894[1], gave the diagram with stator resistance represented and then in 1896 simplified so as to be applicable with ease. His 1894 work was, however, difficult of application and that of Ossanna much simplified this but was still complicated by requiring analytical geometry calculations.

Grob gave his 1901 method of drawing the circle followed by Thomälen in 1903 giving a slightly simpler method and showing the relation to the simple 1896 Heyland circle. Grob in 1904 gave his very much simplified diagram and Thomälen shows that his diagram is applicable.

### Kuhlmann's Diagram

Karl Kuhlmann discusses[2] the general alternating current transformer under three heads "I. . . . without inductive resistance in the secondary circuit (Polyphase induction motor).

"II. . . . with constant secondary phase displacement ($< \phi_2$ constant).

"III. . . . with constant secondary self-induction $L_2$ and variable phase displacement $< \phi_2$, [$\tan \phi_2 = 2\pi\ (Y_1 - Y_2)\ \frac{L_2}{R_2}$] the most general case."

His figure 17, reproduced here as Plate XXII, is based on the case number II. Like Ossanna, he takes currents as being in phase with and of same length as fluxes. That is, he considers OA = primary flux $F_1 = I_1$.

In Plate XXII from similar triangles GE is employed for $I_1$ instead of OA, since $GE = T_1 \times OA = T_1 \times I_1$. If the phase displacement is held constant, the angle FOD is constantly

[1] *Elektrotechnische Zeitschrift*, 1894, p. 561.
[2] *Ibid.*, Apr. 18, 1901.



Digitized by Google

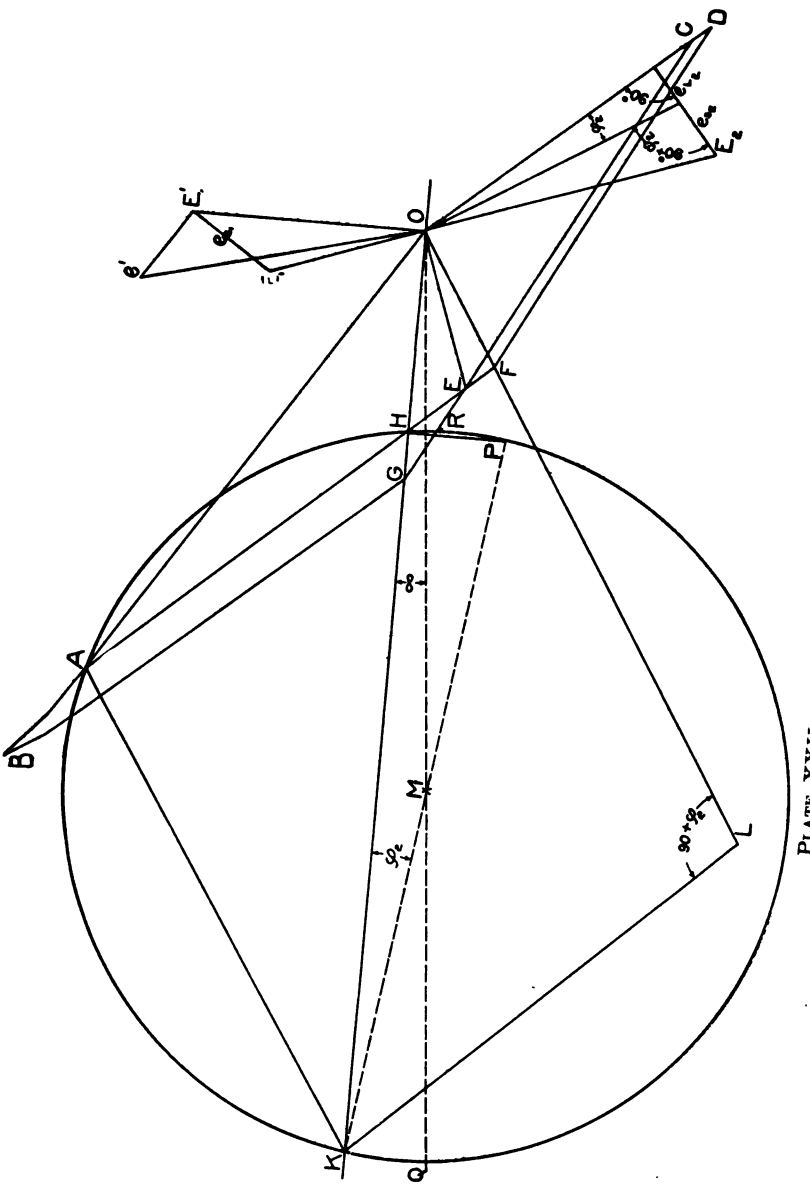



PLATE XXII.—KARL KUHLMANN'S DIAGRAM.



Digitized by Google

equal to $90° + \phi_2$. Since AK is parallel to OF for all loads $HAK = 90° + \phi_2$

$$\frac{OK}{OH} = \frac{LK}{HF} \text{ where } OK = OH\,\frac{1}{\sigma} \text{ and } HK = OH\,\frac{1-\sigma}{\sigma}.$$

Therefore $OK = \frac{1}{\sigma}\,OH$ for any load.[3]

Construct circle M through the three points H, A and K.

$E_1'$ is constant and therefore OH is constant. Draw the diameter KP. Then KPHA is an inscribed quadrilateral and therefore

$$HPK = 180° - (90° + \phi_2)$$
$$= 90° - \phi_2$$
$$HK = PK \sin (90° - \phi_2) = PK \cos \phi_2 = RQ \cos \phi_2$$

Following the secant law

$$OH \times OK = OR \times OQ = \overline{OM}^2 - MQ^2 = \overline{OH}^2\,\frac{1}{\sigma} \text{ but } OA = I_1;\ OH = I_m;$$
$$OM = \alpha;\ MQ = r$$

Therefore

$$I_m^2\,\frac{1}{\sigma} = \alpha^2 - r^2 \text{ and } \alpha = \frac{I_m}{2\sigma \cos \phi_2}\,\sqrt{(1-\sigma)^2 + 4\sigma \cos^2 \phi_2}.$$

In the triangle OAR

$$I_1^2 - 2\,I_1\,\alpha \cos AOR + \alpha^2 - r^2 = 0;\ AOR = 90° - \phi_1 + \gamma$$

in which only $\gamma$ is unknown and it may be found from the triangle HOM.

$$\cos \gamma = \frac{I_m^2 + \alpha^2 - r^2}{2\,I_m \times \alpha} = \frac{I_m^2\left(1 + \frac{1}{\sigma}\right)}{2\,I_m \times \alpha}$$

$$= \frac{1 + \sigma \cos \phi_2}{\sqrt{(1-\sigma)^2 + 4\sigma \cos^2 \phi_2}}$$

$$\sin \gamma = \frac{\sin \phi_2\,(1-\sigma)}{\sqrt{(1-\sigma)^2 + 4\sigma \cos^2 \phi_2}}$$

---

[3] This value of dispersion coefficient $\sigma$ is the same as Grob's T and is equal to $T_1 + T_2 + T_1 T_2$ in terms of Heyland's coefficients. Also equal to $v_1 v_2 - 1$ in terms of Hopkinson's coefficients and equal to $\frac{1}{\sqrt{v_1}\sqrt{v_2}} - 1$ in terms of Behrend's coefficients.



Digitized by Google

The coordinates of the center M are:

$$X_1 = \frac{e_1}{K} \cdot \frac{\beta}{2\alpha} \text{ and } y_1 = \frac{e_1}{K} \cdot \frac{\gamma_1}{2\alpha}$$

$$\text{The radius } R = \frac{e_1}{K} \cdot \frac{1}{2\alpha_1} \sqrt{\beta_1^2 + \gamma_1^2 - 4\alpha_1 \vartheta_1}$$

With $\phi_2 = 0$ these become

$$X_1 = X_0,\ y_1 = y_0,\ R_1 = R.$$

Several other diagrams were investigated but will not be given since they were found not applicable to the work of the second chapter.



Digitized by Google

equal to $90° + \phi_2$. Since AK is parallel to OF for all loads $HAK = 90° + \phi_2$

$$\frac{OK}{OH} = \frac{LK}{HF} \text{ where } OK = OH\,\frac{1}{\sigma} \text{ and } HK = OH\,\frac{1-\sigma}{\sigma}.$$

Therefore $OK = \frac{1}{\sigma} OH$ for any load.[3]

Construct circle M through the three points H, A and K.

$E_1'$ is constant and therefore OH is constant. Draw the diameter KP. Then KPHA is an inscribed quadrilateral and therefore

$$HPK = 180° - (90° + \phi_2)$$
$$= 90° - \phi_2$$
$$HK = PK \sin (90° - \phi_2) = PK \cos \phi_2 = RQ \cos \phi_2$$

Following the secant law

$$OH \times OK = OR \times OQ = \overline{OM}^2 - \overline{MQ}^2 = \overline{OH}^2\,\frac{1}{\sigma} \text{ but } OA = I_1;\ OH = I_m;$$
$$OM = \alpha;\ MQ = r$$

Therefore

$$I^2_m\,\frac{1}{\sigma} = \alpha^2 - r^2 \text{ and } \alpha = \frac{I_m}{2\sigma \cos \phi_2}\,\sqrt{(1-\sigma)^2 + 4\sigma \cos^2 \phi_2}.$$

In the triangle OAR

$$I_1^2 - 2\,I_1\,\alpha \cos AOR + \alpha^2 - r^2 = 0;\ AOR = 90° - \phi_1 + \gamma$$

in which only $\gamma$ is unknown and it may be found from the triangle HOM.

$$\cos \gamma = \frac{I_m^2 + \alpha^2 - r^2}{2\,I_m \times \alpha} = \frac{I_m^2\left(1 + \frac{1}{\sigma}\right)}{2\,I_m \times \alpha}$$

$$= \frac{1 + \sigma \cos \phi_2}{\sqrt{(1-\sigma)^2 + 4\sigma \cos^2 \phi_2}}$$

$$\sin \gamma = \frac{\sin \phi_2\,(1-\sigma)}{\sqrt{(1-\sigma)^2 + 4\sigma \cos^2 \phi_2}}$$

---

[3] This value of dispersion coefficient $\sigma$ is the same as Grob's T and is equal to $T_1 + T_2 + T_1 T_2$ in terms of Heyland's coefficients. Also equal to $v_1 v_2 - 1$ in terms of Hopkinson's coefficients and equal to $\frac{1}{\nu_1 \nu_2} - 1$ in terms of Behrend's coefficients.



Digitized by Google

The coordinates of the center M are:

$$X_1 = \frac{e_1}{K} \cdot \frac{\beta}{2\alpha} \text{ and } y_1 = \frac{e_1}{K} \cdot \frac{\gamma_1}{2\alpha}$$

$$\text{The radius } R = \frac{e_1}{K} \cdot \frac{1}{2\alpha_1} \sqrt{\beta_1^2 + \gamma_1^2 - 4\alpha_1 \vartheta_1}$$

With $\phi_2 = 0$ these become

$$X_1 = X_0,\ y_1 = y_0,\ R_1 = R.$$

Several other diagrams were investigated but will not be given since they were found not applicable to the work of the second chapter.



Digitized by Google

## CHAPTER II

## SELF-EXCITING ASYNCHRONOUS GENERATORS

If an ordinary induction motor is driven above speed by mechanical power, it will operate as a generator and deliver current, provided a leading exciting current is supplied. This leading current may be produced by condenser, by an ordinary synchronous motor floating on the line, by a synchronous generator in parallel, or by means of modifications in the rotor to be described later.

The action of this leading current is to supply a wattless component which shall equal, approach or exceed the wattless magnetizing current.

The problem, then, of making an induction generator self-exciting is reduced to determining some method of supplying this wattless component from the machine itself; and the object of this investigation is to represent on a diagram the action of this exciting current. The historical development of the diagram for the ordinary induction machine has been taken up and several diagrams have been described in Chapter I.

The history of the diagram for the self-excited generator or the compensated motor has been extremely short. It will, however, be taken up and the self-exciting lines developed for the Heyland and the Grob diagram. The different developments which actually give these phenomena will be taken up individually and examined.

The representation on a diagram was first given for the compensated as for the ordinary motor by Alexander Heyland[1] whose work will be displayed shortly. Heubach, De Latour, Bragstad and La Cour seem, with Heyland, to have done most

[1] *Elektrotechnische Zeitschrift*, July 23, 1903.



Digitized by Google

of the work with the actual machines, although Prof. C. A. Adams and Dr. A. S. McAllister in this country should be mentioned.

## Heyland's Diagram for Compensated Motors

Starting with the regular approximate Heyland construction, he briefly runs over the discussion of the diagram and then proceeds to the necessary modifications. The only difference occurs from the fact that the excitation field comes from the rotor, power being led into it from the external supply. This occasions a leakage in the direction opposite to that in the ordinary motor since the rotor has the larger flux.

In consequence of this externally applied e. m. f. in the rotor added to the electromotive force induced by its motion in the stator field, the secondary current no longer stands at a right angle to the rotor or secondary field, but makes a smaller angle with it, Plate XXIII.

In the current triangle we will have as before AC as the wattless magnetizing current of the stator field and $AC^k$ the magnetizing current of the rotor field; AC being constant and $AC_1^k$ parallel to the rotor field.

To determine the rotor current we must obtain the components of the e. m. f. which produces it and combine them. That produced by the motion in the stator field $e^l$ will vary in size with the slip and will always be perpendicular to the rotor field; the other $e^k$ in the most general case of the application of a constant line pressure will have a constant length and a direction dependent upon the degree of brush displacement, here called the angle $\alpha$.

This gives a figure with AC = ADT all being constants, $e^l$ perpendicular to CK′D and $e^k$ constant in length and direction, given.

As in the ordinary motor the corners of the triangle move on circles which may be determined as follows: Call the angle of brush displacement zero when the pressure impressed on the rotor is in line with the stator field. Any other brush angle



Digitized by Google

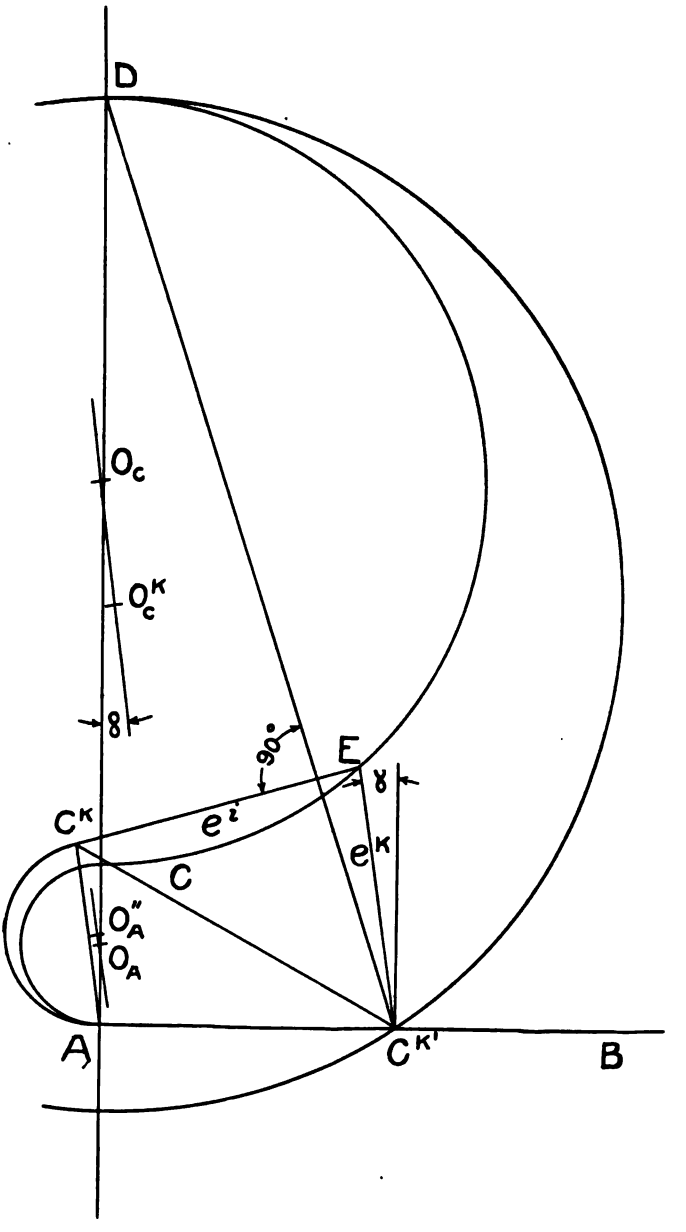



PLATE XXIII.—DIAGRAM COMPENSATED MACHINE.

[472]

Digitized by Google

will be the angle $\alpha$ and is measured from the direction of AD. Lay off from $O_c$ at the angle $\alpha$ a length $O_cO_c^k = \frac{e}{2}(1 - T)$, and from $O_a$ at angle $\alpha$ a length $O_aO_a^k = \frac{e}{2} T$.

Draw a circle with $O_c^k$ as center to pass through D; draw another with $O_a^k$ as center to pass through A. The construction of the torque and mechanical power circles and of the slip line is made in the same way as for the ordinary motor.

This diagram yields a clear idea of the working of compensation in the motor, for, let the compensation be zero, $e^k$ becomes zero. $O_c^k$ falls back $O_c$ and we have the original diagram. The greater the compensation the larger diameter has the circle and the more is the power output, (see later reference to an Observation of Heyland's ETZ 5/28/03). For but one load is the power factor exactly unity and in order to obtain this the no-load current must be a leading one, that is, the machine must be over-compensated.

Losses in the compensated are less than in the ordinary motor, the iron loss in stator is the same since it arises from the same stator field, while the rotor iron loss is slightly greater, due to greater flux but still negligible, while the copper losses in stator are smaller since for a given output the current is less with increased cos $\phi$ and similarly in the rotor from a certain load upwards the loss is less than in the ordinary motor.

This he shows[2] by data experimentally determined in a machine with a stator and two rotors, the first one employed being compensated, the second a wound rotor with slip rings. Measuring the rotor current revealed the fact that only on very low loads was the $I_2$ of the compensated greater than that of the ordinary motor. The explanation comes from the fact that power output equals common field times induced rotor current.

Since the flux is greater in the compensated motor to produce the same power output, a less current is needed in the secondary, or, to express it differently, for the same rotor current and therefore heat-loss the power output will be larger than in the ordi-

[2] *Ibid.*, May 28, 1903.



Digitized by Google

nary motor. This effect is sufficient to more than allow for the increased current in the rotor due to compensation. He gives the following table:

| | Normal motor. | Compensated Motor. | |
|---|---|---|---|
| | | Same power. | Same heating. |
| H. P. | 5 | 5 | 7 |
| Iron and friction, per cent. | 14 | 14 | 10 |
| Slip, per cent. | 4 | 2 | 4 |
| Stator copper loss, per cent. | 4 | 2 | 3 |
| Rotor copper loss, per cent. | 4 | 3.5 | 6 |
| Per cent. efficiency | 78 | 80.5 | 81 |

To return to the diagram, we see that the slip is for any load smaller than in the ordinary motor and varies inversely as the rotor current.

A figure (Plate XXIV) gives a very interesting comparison of the cos $\phi$-output curves of the ordinary motor and of the compensated at various brush settings. As is noticed, the power factor rises much more quickly, attains and passes through unity but hugs it much more closely than does the ordinary motor going also to a greater output before it turns back and the output decreases. The greater the angular displacement, the quicker the power factor rises and the greater the output. It is found, however, that large brush displacement influences the efficiency unfavorably and this puts an effectual stop to too great a displacement.

As will be readily noted, the brush position which gives the best cos $\phi$, at full load under motor action gives too small an output when the machine is run as a generator. Since there can never be inductive lagging power factor and any output from the machine, the output is limited by the intersection of the circle with the pressure line. This will be largely raised by increasing the brush displacement.[3]

[3] This does not mean that an inductive load cannot be supplied, but simply that there must be enough leading current to prevent the total power factor being a lagging one.



Digitized by Google

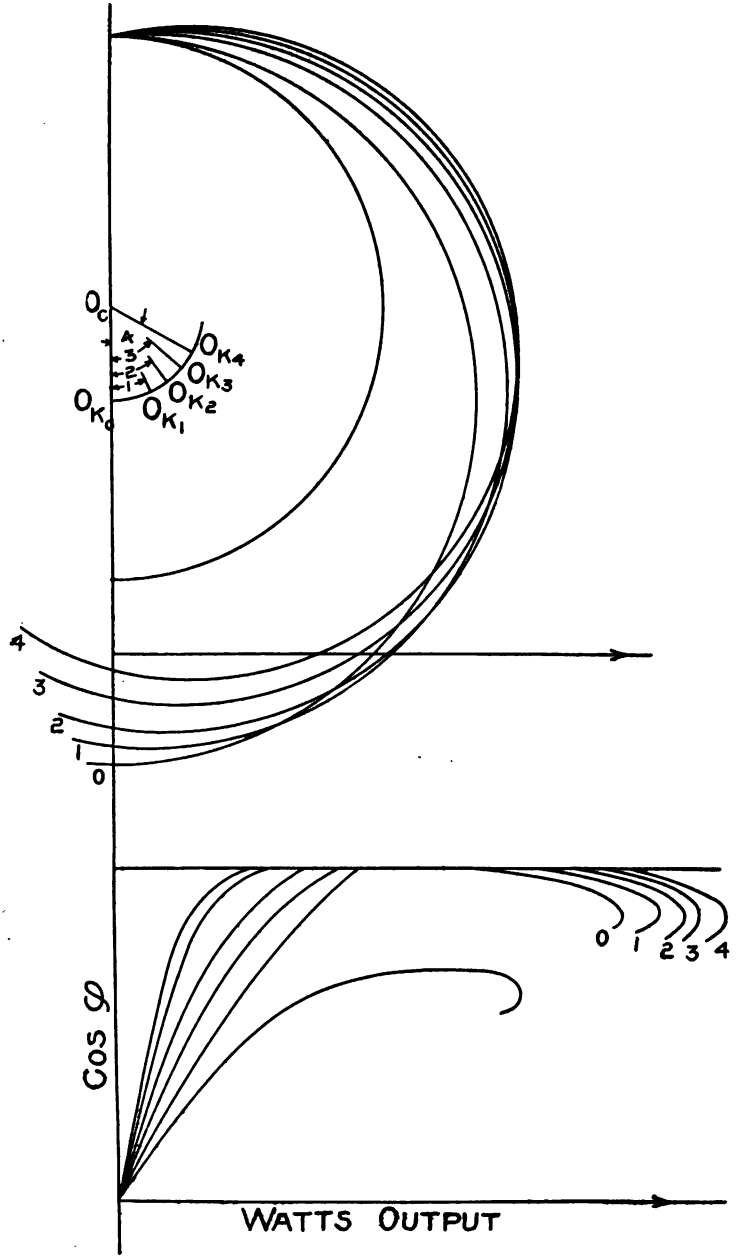



PLATE XXIV.—HEYLAND, July 23, 1903.

[475]

Digitized by Google

The use of some motors over-compensated on light load upon the same mains as some regular induction motors with lagging cos $\phi$, should be noted as an excellent and simple method for improving the general power factor of a system.

In chapter I, reference was made to the theory of the induction machine built up by Julius Heubach in his book, *Der Drehstrommotor* (Julius Springer, Berlin, 1903). In 1904, in the *Vorträge,* Heubach gives a *Theorie der Kompensierten Asynchronmaschine,* in which he develops working basis for the compensated machine along the same lines as in his book for the ordinary induction motor. He deals first with an ideal motor and gradually introduces the complications which are really met with, dealing, however, almost entirely with the motor and saying that the generator action may be obtained by completing to circles the five semi-circles which are the loci of the ends of the current pressure and flux vectors.

He offers no experimental verifications, nor has he any easy method of construction or reading off of the mechanical and other characteristics. Notwithstanding, however, his work is of a very high standard and is a noteworthy contribution to the literature. Since his method does not obviate the experimental errors in the determination of the various leakage factors, the results to be obtained from his diagrams are necessarily limited by the large per cent. of inaccuracy which is the best available in determining these leakage values. One reason why Grob's diagram stands preeminent is that instead of necessitating the observation of these various leakage factors, the data to be taken are easily and directly read and if desired the leakage factors may be deduced from the circles with a very considerable accuracy.

## Oscillograms

The theory assumes that pressure, current, and flux are all sine waves, and it is of interest to record the wave shapes used in taking the experimental data. The first data on generator test were taken with a pressure wave like No. 1, Plate XXV, which yields when subjected to the 80 part Harmonic Analyser



Digitized by Google

of the mathematics department of the University of Wisconsin, the Fourier series,

$$57 \sin x - 8 \sin 3x + 17 \sin 5x \text{ (irregular)}$$

The current waves will be distorted as shown in Nos. 5 and 6 in the well-known way produced by a circuit containing inductance and resistance, the current wave being as shown practically independent of the shape of the pressure wave.

Pressure wave No. 2 was employed in the second set of generator experiments since it is much nearer the desired sine wave giving the following series.

$54 \sin x - 6 \sin 3x$

Oscillograms Nos. 3 and 4 are from the induction generator, capacity excited by condensors and thus free from the influence of any impressed wave shape. No. 3, the pressure wave at no load, is a very smooth curve with the formula:

$6 \sin x - 5 \sin 3x$, other terms negligible,

a very good approximation to the sine curve.

The current wave of No. 4 was taken when the current per $\phi$ was 18.3 amperes, the pressure 109, and the power factor $\cos \phi = .904$. The less apparent displacement of the oscillogram is without doubt due to the fact that the vibrator of the oscillograph was shunted across a lead through only a small part of which the total current flowed, much of the resistance drop being due to a current supplying lamp load only.

In connection with this curve, a rather peculiar phenomenon was observed. Superimposed upon the main current wave (appearing stationary in the oscillograph) was a ripple which appeared periodically and traveled from left to right over the wave, making difficult the accurate determination of the main wave. The period of recurrence of this traveling wave was rather long and its wave length much shorter than the main wave. This may be caused by the fact that, since the numbers of the stator and rotor teeth are made to have small greatest common divisor, there are only rare intervals when the flux path is highly good and the flux passes in a stronger tuft. If the rotor were running in synchronism with the stator frequency, these tufts would show up on the pressure wave also.



Digitized by Google

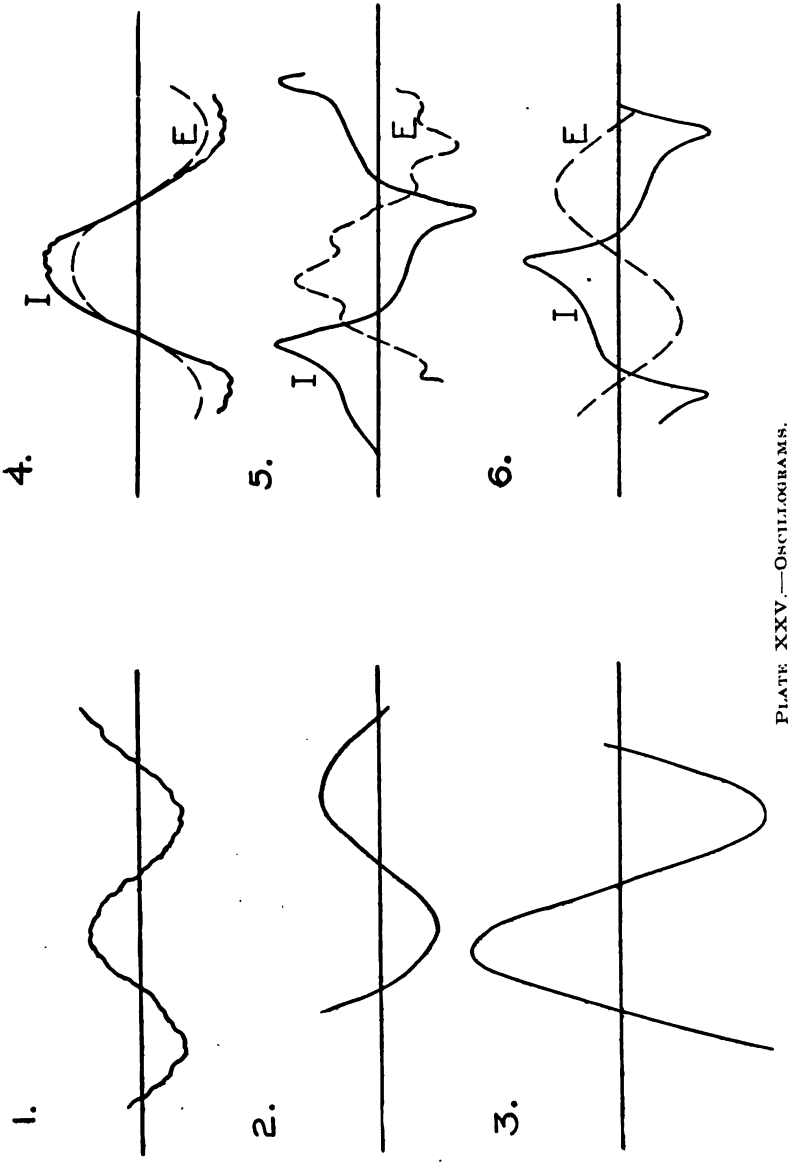



PLATE XXV.—OSCILLOGRAMS.



Digitized by Google

Since, however, there is a slip, the recurrence of this bunch of lines of force takes place only at a rate much slower than that of normal frequency, giving, in fact, a traveling wave whose rate of recurrence is equal to the slip. This is a rather interesting carrying through of stroboscopic revealing of hidden phenomena.

The oscillograms listed are

No. 1. No load $3\phi$ pressure from a $3\phi$ and $2\phi$ synchronous machine. Wagner No. .919, 10 H. P. Type 6.

No. 2. No load pressure from a $3\phi$ synchronous machine G. E. Co. No. 75654, 16 H. P. Type A. S. B.

No. 3. No load pressure from $3\phi$ Induction Generator, Brown, Boveri No. 193913, 6 H. P. Condenser excited.

No. 4. Load pressure and current for No. 3 generator.

No. 5. Pressure and current on load composed of L and R in series. From one of the city supply generators.

No. 6. Same curves as No. 5 from a $3\phi$ synchronous machine. National No. 1114. Type R. B. 200 kw.

In connection with the operation of the induction generator excited by condensers in the high pressure side of transformers in delta across the stator, (See Plate XIX) a rather interesting phenomenon was noted. As pointed out by A. S. McAllister, an induction generator will rise to the voltage represented by the intersection of the magnetization curve, plotted between volts and exciting current, and the line drawn to represent the condenser used to excite the machine. He also pointed out that there would be some capacities so low that they would nowhere intersect the curve and therefore give no pressure as A. Increasing the capacity increases the pressure as shown. If the capacity increases indefinitely, the pressure would theoretically soon come to a practically constant value and remain there, demanding a very much greater magnetizing current, provided the frequency remains constant. The limit of this action is when an infinite capacity, i. e., a conductor, is shunted across the stator. This infinite capacity line would cross the magnetization curve at $\infty$ and o giving maximum pressure with $\infty$ magnetizing current or zero pressure and current. As seen by ex-



Digitized by Google

periment when an infinite capacity, in this case a severely punctured condenser, was put across the line the excitation was entirely lost. Ordinarily, when the circuit to an induction machine is broken, magnetism must be left in at least two of the three phases, no matter at what point of the wave the switch is pulled, and, upon again starting as a generator the machine will pick up by itself. After passing through the reduction to zero, however, by means of $\infty$ capacity all the magnetism is killed, there is no residual, and one phase must be primed from an external source in order to start the machine. In some cases when the punctured condenser was inadvertently left included, the machine would not excite even though primed, since it immediately returned to zero.

Heyland in describing some tests on his first machine says in this respect: "The chief difference in the asynchronous alternate current dynamo is the fact that it does not possess fixed poles as in the continuous current dynamo and *has therefore also no residual magnetism.* . . . residual magnetism is absent and after bringing the generator up to speed it must be separately excited from some external source."[4]

The machine which I used was regular stator construction without pole-pieces and yet picked up invariably, unless the magnetism had been killed by raising the capacity to an infinite value. It would appear that Mr. Heyland's machine should have picked up also. The phenomena attending critical speed were also observed. That is, there was a definite speed below which the machine would not pick up with a certain condenser and above which it worked nicely. It was noticed that the change of the capacity in any one leg influenced the voltage of the whole system so the experiment was tried, keeping the speed of prime mover constant, of adjusting the pressure with varying load by means of a variable condenser in one leg. It behaved admirably, acting, of course, simply as a field rheostat in a shunt direct current machine to vary the amount of the exciting current. With constant rotor speed, however, the increased slip

[4] *Electrician* (London), Jan. 24, 1902.



Digitized by Google

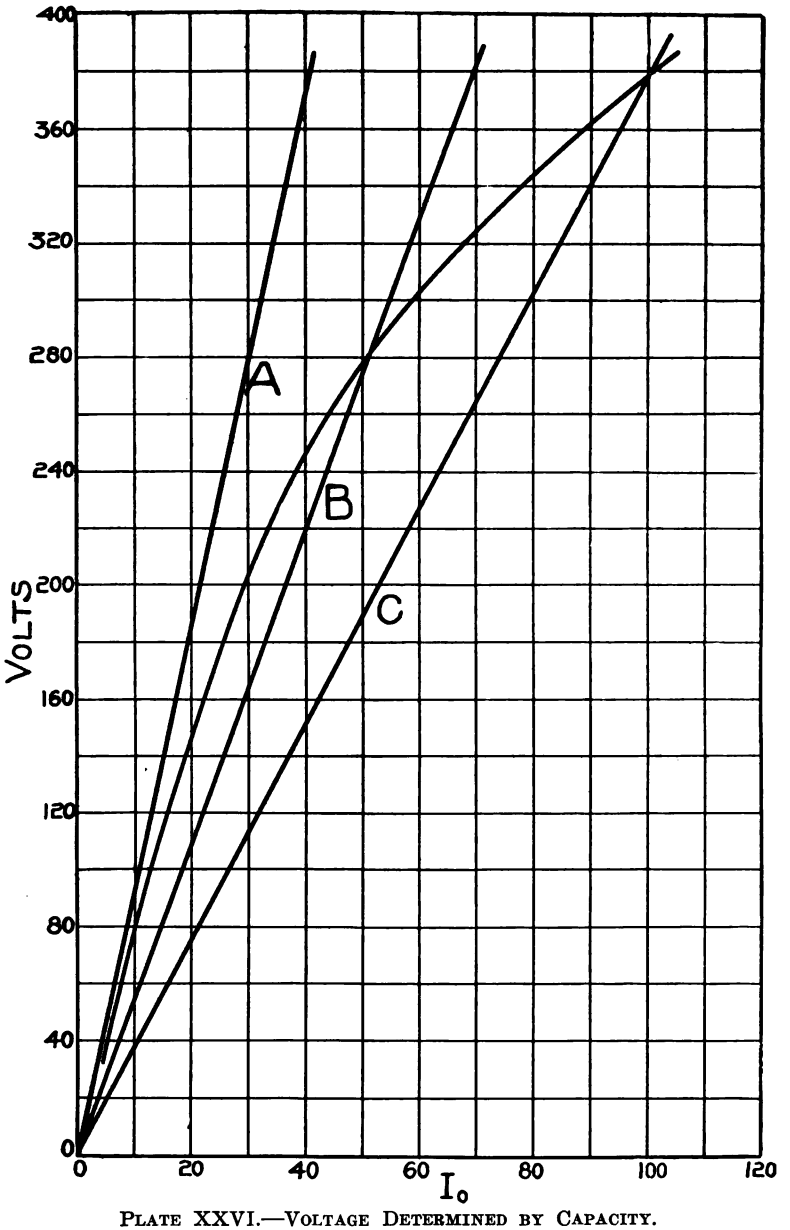



PLATE XXVI.—VOLTAGE DETERMINED BY CAPACITY.



Digitized by Google

under load necessarily gives a lowered line frequency; this lower frequency applied to the condenser requires for the same effective capacity-reactance a much larger capacity in microfarads than as though the line frequency were constant. This means that the regulation is very bad and with increasing load the attendant in charge of the condenser must be very active or his voltage will not only fall low but he will lose his excitation and have to begin anew. The curves, shown in Plates XXVII and XXVIII, are very ragged, following the variations in pressure. These, however, are as closely adjusted as the steps of the condenser would permit.

The lines for the compensated motor in the Grob diagram may be found as follows. In the Heyland diagram the secondary power factor is taken as unity, that is, secondary current $I_2$ in phase with $E_2$ and therefore at right angle to $\phi_2$, the secondary flux. This cannot be true for any position except at synchronism where the frequency in the secondary circuit is zero, since there is some inductance even in a squirrel-cage rotor, and L combined with even a small f will inevitably produce its $2\pi$ fL and phase angle.

This angle will increase up to standstill where full frequency is impressed; in fact, in both directions from synchronism the frequency will increase until infinite frequency is reached; but by what law this variation occurs is not known. Some preliminary experimentation showed that this determination of law of change of cos $\phi_2$ is far from free of complexities and, due to shortness of time, this particular line of investigation was abandoned for the time (although the author hopes later to take this point up much more fully) when it was found that variations in pressure much smaller than could be regulated varied the observations to such an extent as to make consistent data impossible.

The problem even for the ordinary motor is an interesting one, for we have in addition to the increasing frequency with increase of load the increase of saturation; and varying with that the increase of current, and flux—wave-distortion, which though small at no load due to the air-gap, soon becomes far from negligible. $L_1$ and M both vary with load also and like so many



Digitized by Google

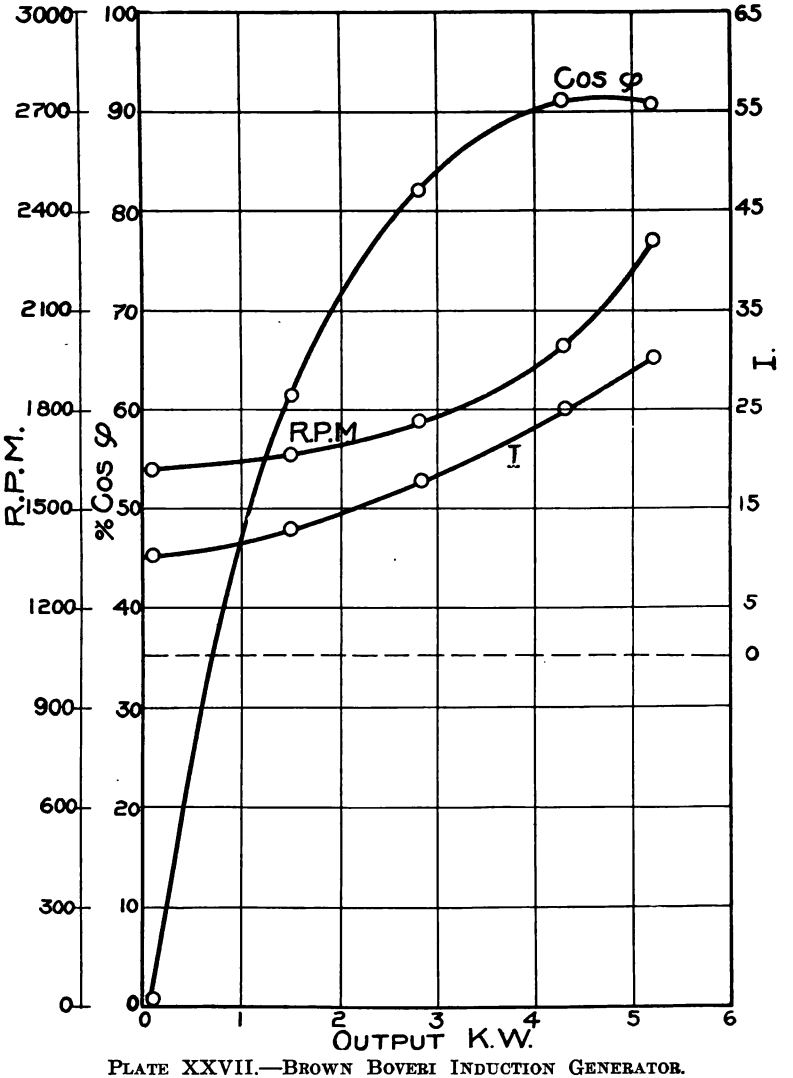



PLATE XXVII.—BROWN BOVERI INDUCTION GENERATOR.
Capacity excited.



Digitized by Google

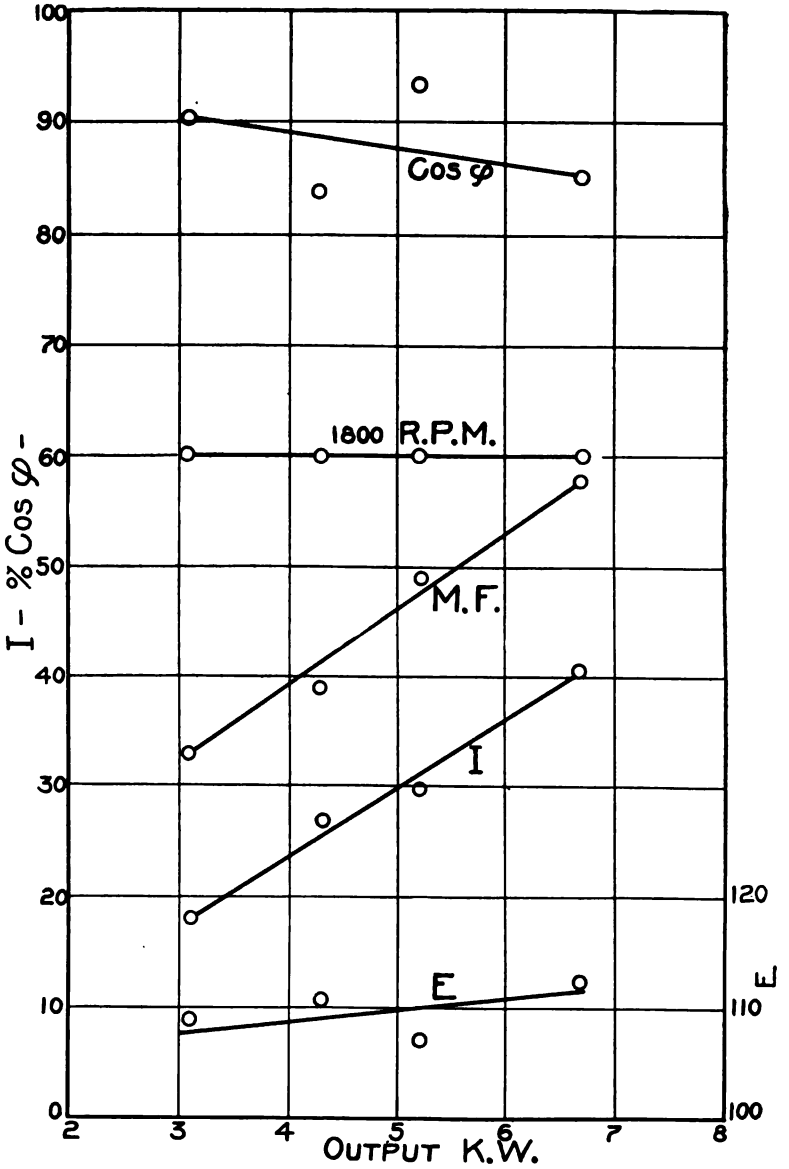



PLATE XXVIII.—BROWN BOVERI INDUCTION GENERATOR.
Capacity excited. Constant speed.



Digitized by Google

of the functions in the motor with a most peculiar law of variation.

On the whole, to quote a member of the department of physics, "this point alone would make a most excellent and complete subject for an investigation for a Ph. D. thesis."

Acknowledging, then, the source of error involved in the compensated-motor diagram for which this is in a measure an underlying point, I proceed to the development, claiming, however, as is due, a much greater accuracy than is found in Heyland's. The limits upon the circle between which the free end of the secondary flux vector can move are a very small per cent. of the total circumference and thus the total error is not large.

Starting, then, with the Brown, Boveri motor, on which the most accurate work has been done here, let us consider that it is fitted with a direct current armature with shorted commutator and that the brushes are displaced five electrical degrees. (Plate XVI.) Let us further assume that we desire to obtain unity power factor at a load a little less than full load. The construction will proceed as follows:—parallel to the diameter through the no load current position, draw a line through the desired value on the pressure line (where, of course, $I_1$ and $E_1$ are in phase at unity cos $\phi$).

Lay off an angle of 5° from this line and another at 5° from the center of the circle. Draw the flux line and perpendicular to it, through the intersection of the 5° line and the circle, a line to intersect the secondary circle. Draw a line from the new primary current end through the upper intersection of the circles; this is the new secondary current. The change in the secondary circle is so slight as to be inappreciable, the increase of secondary current coming at the other end. Now, with center on 5° line through the center of the original circle draw a circle through the end of the new primary current and through the far end of the flux line. This will be the circle for the compensated motor and self-excited generator under the conditions specified and to the same scale as the ordinary circle. The maximum motor output will exceed that of the uncompensated motor and the power factor will be much higher starting as a leading—passing through unity and continuing as a lagging-power factor. At the maxi-



Digitized by Google

mum output (about 15.6 h. p. with this arrangement as compared with 12.3 h. p. as a normal motor) the power factor is still as high as 91 per cent. As a generator, the machine will be self-exciting only so long as the circle stays on the right of the pressure line. With this particular brush position the output could rise only to about 6 h. p. This shows the necessity of a greater brush displacement for generator than for motor action and also that for best performance the brushes must be shifted when the change in function occurs. This is true since the position which would give proper output as a generator (a larger diameter circle) would give a motor power factor very bad as a leading current. To represent different brush displacements other circles are necessarily constructed in a manner just like the above except that some other number of degrees is selected and the new center is differently displaced. If it is desired to have unity power factor fall at some other load it may readily be done simply by choosing that point as the origin of field of flux, and of secondary current. As has been pointed out, it is entirely possible so to adjust a compensated motor that its leading current at light loads will just balance the lagging currents of similar uncompensated motors at light load. Curves have been drawn for the ordinary motor and generator and for the same machine compensated for unity power factor at normal full load and with 5° brush displacement.

As plotted, the curves for the 5° machine seem to point to error, since not alone is the ratio of mechanical to e. h. p. greater than one, during much of the range, but there is a time when both are output. It must be remembered, however, that the e. h. p. curve involves only the power brought to or taken from the stator and to represent the total this would need to have added to it the power to the rotor. The most interest attaches, however, to the relation between the stator currents in the two cases. The appearance of mechanical power from the rotor and electrical at the same time from the stator is like taking, in the ordinary motor, current and mechanical power both from the rotor. The machine acts in each case both as motor and as transformer being thus Steinmetz's "most general case of the alternating current transformer" since it gives out part of the energy



Digitized by Google

as mechanical power and part as electrical at changed pressure and frequency. As shown, (Plate XVIIb) the compensated machine will give motor output nearly up to synchronism with input to both stator and rotor. Then the rotor becomes the only point of input and mechanical power and electrical from stator may be obtained up to a little *above* synchronism. Thereafter, mechanical power must be input and electrical output is derived from the stator. Self-excitation with 5° displacement continues only up to a trifle over 6. h. p. and thereafter some other source of leading current must be employed, although, as will be noted, the amount of this leading current is much less than is required by the normal motor.

It will be noted that while the power factor rises both in motor and generator much faster and higher than when uncompensated, although it reaches unity and passes through, it merely hovers near it and does not, as Heyland's early articles maintained, give "unity for all loads." It does stay good up to more than 100 per cent. overload as a motor (as a generator it soon falls off), but not at unity, and while .97 is good, it still represents an angle of 14° 4′ between pressure and current, while .95 is 18° 12′ and .90 (often reached by ordinary motors) is only 25° 50′, an angle of 8° 6′ with .99 power factor.

The curves of primary current for the two machines are much alike; the main difference being in their minima, the compensated falling much below the ordinary induction motor for its minimum current when power factor equals zero.

An efficiency curve can, of course, not be plotted for the compensated machine until the total input is known both to stator and rotor.

The rotor current flows due to a pressure which is the resultant of two components, one due to motion in stator field which is perpendicular to rotor field and varies directly with the slip, the other component is that which comes in from the line and is of constant value and is fixed in position by the degree of brush displacement.

At synchronism the first of these falls to zero since there is no relative motion and therefore no cutting of stator lines by the rotor. The current triangle falls into a straight line at syn-



Digitized by Google

chronism in which case the equation $I_2 = I_1 + I_m$ holds algebraically, whereas in all other places it is merely a vector equation.

To determine rotor input for positions other than synchronism a knowledge of secondary power factor is requisite and since this is a variable of unknown law this solution will have to be postponed for some future investigation.

BROWN, BOVERI COMPENSATED 5° DISPLACEMENT

Scaled from diagram. Plate XVI.

| Slip | $I_1$ | STATOR H. P. | | Per cent Cos $\phi$ |
|---|---|---|---|---|
| | | Electrical. | Mechanical. | |
| ∞ | 188 | 11.8 | 19.3 | 24.0 |
| Standstill. | | | | |
| 1.00 | 166 | 23.8 | 0. | 55.0 |
| .761 | 150 | 26.2 | 7.1 | 67.0 |
| .470 | 125 | 26.0 | 13.3 | 79.7 |
| .311 | 100 | 23.2 | 15.4 | 89.0 |
| .229 | 80 | 19.6 | 15.2 | 94.0 |
| .192 | 70 | 17.5 | 14.4 | 96.0 |
| .163 | 60 | 15.2 | 13.5 | 97.7 |
| .143 | 50 | 12.8 | 12.0 | 99.0 |
| .109 | 40 | 10.4 | 10.4 | 99.8 |
| .078 | 30 | 8.0 | 8.5 | 100.0 |
| .052 | 20 | 5.2 | 6.4 | 99.0 |
| .035 | 15 | 3.8 | 5.2 | 97.8 |
| .029 | 10 | 2.6 | 3.8 | 93.4 |
| .007 | 3.7 | 0. | 1.5 | 0. |
| .000 | 4.0 | .4 | 1.0 | 39.0 |
| .005 | 6.3 | 1.4 | 0. | 82.0 |
| .010 | 10 | 2.6 | 1.2 | 96.2 |
| .016 | 15 | 3.8 | 2.6 | 98.9 |
| .023 | 20 | 5.2 | 4.2 | 99.9 |
| .028 | 24 | 6.1 | 5.2 | 100.0 |
| .036 | 30 | 7.6 | 7.1 | 99.9 |
| .049 | 40 | 10.2 | 10.1 | 98.9 |
| .062 | 50 | 12.7 | 13.4 | 97.6 |
| .075 | 60 | 14.9 | 16.5 | 95.9 |
| .102 | 80 | 18.9 | 22.7 | 91.3 |
| .131 | 100 | 22.2 | 29.1 | 85.1 |
| .178 | 125 | 24.4 | 36.6 | 75.0 |
| .243 | 150 | 23.7 | 42.1 | 61.0 |
| .400 | 175 | 18.2 | 44.7 | 39.0 |
| .506 | 185 | 13.0 | 40.9 | 23.4 |
| 1.000 | 192 | 0. | 33.0 | 0. |
| ∞ | 188 | 11.8 | 19.3 | 24.0 |

## THE MACHINES

In the production of a machine which will excite itself as a generator and have a good power factor as a motor, many hybrids have resulted some of which are important and others mere freaks.



Digitized by Google

The general scheme has been to place in the rotating field of an ordinary stator a rotor which consists essentially of a direct current armature, (modified in different ways to suppress sparking, etc.), into which current of the frequency of the line is led through brushes spaced around the commutator at electrical angles equal to the number of degrees of the system, 90° for two phase, 120° for three phase, etc.

The first schemes merely short circuited the commutators of ordinary direct current armatures through a low resistance, while the later ones have produced marked changes in the internal construction and winding, all retaining, however, the commutator and three brushes, which by their displacement determine where in space the exciting field, now coming from the rotor, shall be held fast.

Since compensation brings the slip to lower values, making more care requisite in paralleling though not needing synchronization, there have resulted as examples of the hybrids mentioned above, asynchronous motors which run at synchronous speed, and synchronous motors with a pronounced slip.

Any device by which we can produce the wattless component necessary for excitation, on the scene of action, and obviate its transmission with the accompanying lowered power factor, will be an immense improvement.

If with an ordinary induction rotor locked in position current of the line frequency be led to the slip rings, a revolving field will be set up in the rotor which will travel around at the same rate as that in the stator, and, by making its ampere-turn strength exceed that of the field by the proper amount, the wattless component of the stator current may be annulled. If the rotor be brought up to synchronous speed, the same results can be achieved by using a direct current of the proper value. (This becomes an induction motor running synchronously.)

For speeds other than standstill and synchronism the wattless primary current may be balanced and the line power factor raised to unity by impressing the proper frequency on the rotor.

This proper frequency may be obtained by supplying a commutator and brushes to which line frequency is led. No matter at what speed the armature may be rotated the commutator



Digitized by Google

causes the current to pass through conductors having the same position in space and therefore the same relation to the magnetism of the stator.

The frequency necessary at any time is line frequency less the frequency of the rotation. Thus at standstill 60 frequency —0 frequency = 60 frequency. At synchronism 60 frequency —60 frequency = 0 frequency; at half speed 60 frequency— 30 frequency = 30 frequency, etc.

An ordinary direct current armature would have no torque since no current would flow in it. We must, therefore, connect together points which have some difference of potential that current may flow and torque be exerted. To this end Heyland in his first machine simply shorted the whole commutator. (This was also done at practically the same period by Professor D. C. Jackson at the University of Wisconsin.) These shorting resistances form a shunt through which the reactance voltage at the instant of commutation may discharge and thus improve sparkless conditions at the brushes.

As to the degree to which commutation has been improved opinions differ. Most of the foreign comments are decidedly favorable, but as most of the writers were describing the machines they themselves had invented, we must remember that their panegyrics may, in a measure at least, be due to enthusiasm. In this country, Prof. C. A. Adams' experiments with a G. E. experimental model did not show perfection, though he admits his connections (for compounding) were a little questionable. Dr. A. S. McAllister in his most excellent treatise, *Alternating Current Motors*, says (p. 45): "So effective is the elimination of sparking that no observable result is produced by shifting the brushes into any position." Also: "The resistance forms a non-inductive shunt . . . which fact allows the use of copper brushes of cross section much less than that required for carbon brushes and the commutator is given a correspondingly smaller size." Prof. Adams says: "The large exciting currents used in the generator tests gave trouble on another score, that of sparking; in fact with any exciting current within the working range there was some sparking, but with the larger currents the sparking was so vicious as to require that the com-



Digitized by Google

mutator should be turned down after a run of a few hours. It is thus evident that the damping effect of the low resistance squirrel cage is not sufficient to warrant the use of copper brushes, certainly not with such a small number of commutator bars.''[5] (The squirrel cage mentioned is another device to obtain torque and supposedly to prevent sparking.)

Like so many other points, in a subject of this nature so far from being settled, it is necessary to weigh the evidence, in the absence of personal experimentation, and then draw your own conclusions.

The first motor was Görges,[6] a series polyphase motor, which was excellent at synchronous speed but execrable elsewhere and since the series motor characteristics do not leave it long at synchronous speed it met with no success.

Blondel[7] brought out a motor which was in a measure the complement of Heyland's since it consisted of the same stator and same short-circuited direct current armature but used in place of the polyphase rotor excitation, direct current excitation put into the armature through brushes rotated by a synchronous motor. The action of this motor was successful, it being in all essentials identical in theory with Heyland's, but the complication of the synchronous motor and keeping the pressure of brush on commutator uniform prevented commercial success.

It is regrettable that so many additions to knowledge are accompanied by discord. The compensated motor is not an exception; the discussion between Heyland and Latour as to priority of invention being almost as acrimonious as the Tesla-Ferraris controversy. Heyland's first article[8] appeared in 1901 and described the motor with direct current armature shorted through non-inductive resistances; and also the machine with two windings, one straight direct current, the other squirrel-cage. He noted that it would self-excite as a generator.

Latour describes[9] a very similar machine intended as a gener-

[5] *A. I. E. E. Trans.*, **20**: 780.
[6] *Elektrotechnische Zeitschrift*, Dec. 25, 1891.
[7] *Ec. El.*, Aug. 29, 1890.
[8] *Elektrotechnische Zeitschrift*, Aug. 8, 1901.
[9] *Industrie Electrique*, Feb. 25, Apr. 10 and 25, 1902.



Digitized by Google

ator principally. It differs from Heyland's in having no squirrel-cage and is in fact as he calls it a "shunt-machine." The current of slip frequency finds a path, in lieu of any other, back through the supply lines.

Osnos has described[10] a machine in which the exciting current is commutated in a small auxiliary armature and then led in rotor through rings.

At the St. Louis Congress, where the compensated machine was severely criticized as being beautiful but not applicable to machines of sufficient size to be commercially practicable (500 Kilovolt-ampere machines have been built and run successfully), this device of external commutator was spoken of as the possible saving of the situation for the compensated machine. In all of Heyland's numerous machines this commutator (called "sifting device" by Prof. Karapetoff since it separates watt from wattless current and allows of real flat compounding regardless of power factor) is the one unchanged and unchangeable feature.

An alternator with a practically constant frequency and speed but requiring no synchronizing to parallel, which can be compounded to any desired extent and is absolutely self-contained except for static transformers, being thus independent of any exciting source whatsoever, is assuredly a desideratum. The statement made at St. Louis that in any plant large enough to need such regulation there would always be an attendant who could watch the voltage, does not appeal to me as at all removing the desirability of an absolutely *automatic* regulation. A synchronous alternator with a good Tirrell or similar regulator in the exciter circuit would be hard to beat on load of unity power factor but there is no other device than the compensated and compounded induction generator employing this sifting commutator which will automatically compound for the energy current alone in circuits of any power factor.

The figure (Plate XXIX) shows two rotors both essentially direct current armatures but modified; one by resistances shorting from one commutator bar to the next, the other by the ad-

[10] *Elektrotechnische Zeitschrift,* Oct. 16 1902.



Digitized by Google

dition of a regular squirrel-cage winding of medium resistance in the same iron with the direct current winding.

The upper figure (Plate XXIX) shows a favorite connection which obviates the expense of a separate transformer for reducing line pressure to a value which can be successfully handled by the rotor commutator. As seen, it employs the stator iron as core for a polyphase transformer of which the normal stator winding is primary and the additional low tension winding shorted through the rotor, which may be of any type, is the secondary.

Heyland's most successful winding scheme is shown by the upper pair of figures which picture a two-pole three phase machine showing two conditions in the commutation cycle. The brushes are shown as moving, the armature stationary, of course identical in result with the actual condition since relative motion alone is essential. The winding shown has 2 bars per pole per phase that are active, and 1 per pole per phase inactive, thus making 18 in all. The winding is in six sections shorted inside the bars by 10 ohmic resistances by the proper proportioning of which the reactance voltage of commutation may be reduced theoretically with a sine wave to zero. Practically there exist enough harmonics in every wave to produce some sparking at any time, but a small minimum may be reached and it is interesting to note, as pointed out by Heyland's paper at St. Louis, that it is possible to reduce to zero with a perfect wave, a thing not ever possible with a direct current commutator.

The brushes are made wide enough to span the inactive segments as shown to the right where brush I is in the act of commutating, that is, short circuiting, the active segments of opposite sign. In these two figures is shown the action of this machine which is a successfully self-exciting and compounding *synchronous* generator. Though not coming strictly under the heading of this thesis this machine is included because it is a direct product of work on asynchronous machines and would never have been evolved save as the limit of gradual injection into the induction machine of characteristics of the synchronous. It becomes very difficult to tell to which class a machine really



Digitized by Google

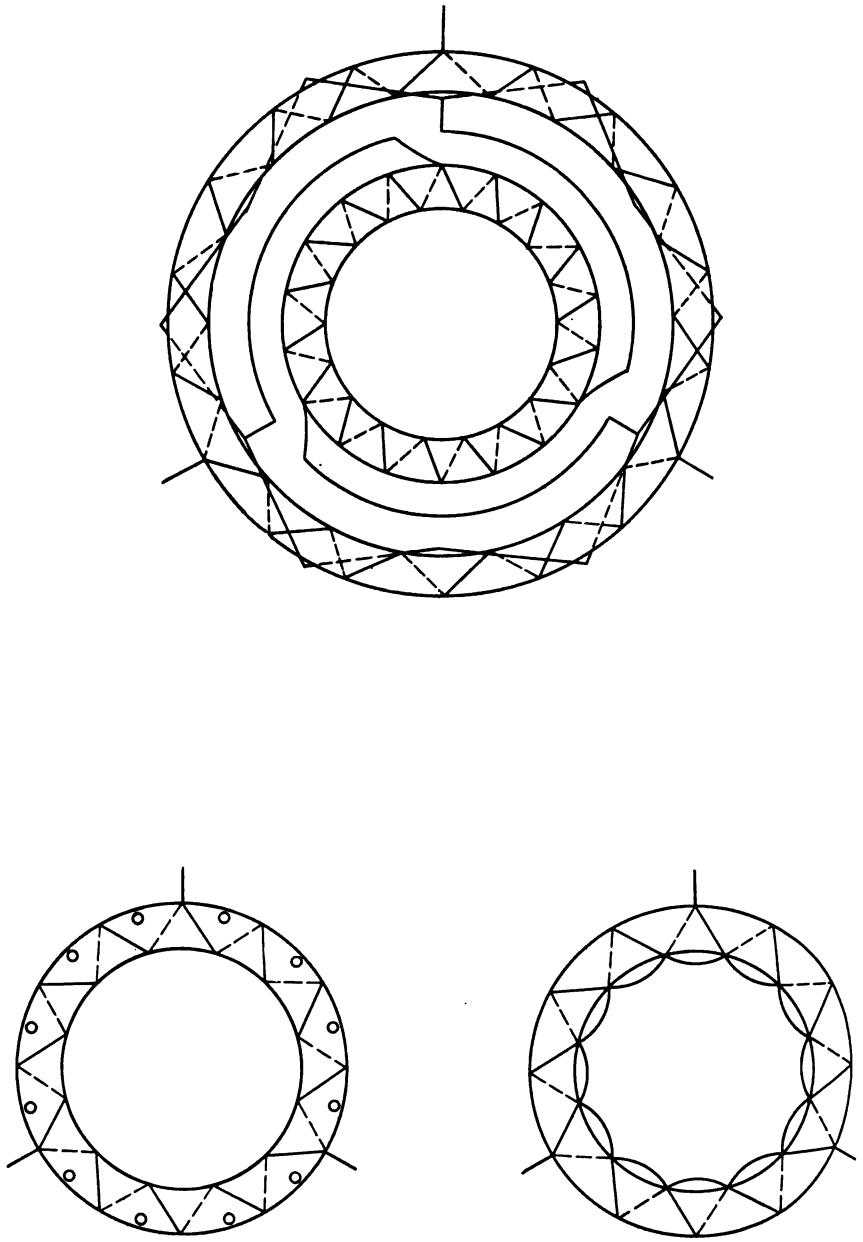


PLATE XXIX.—WINDING DIAGRAMS.
[494]

Digitized by Google

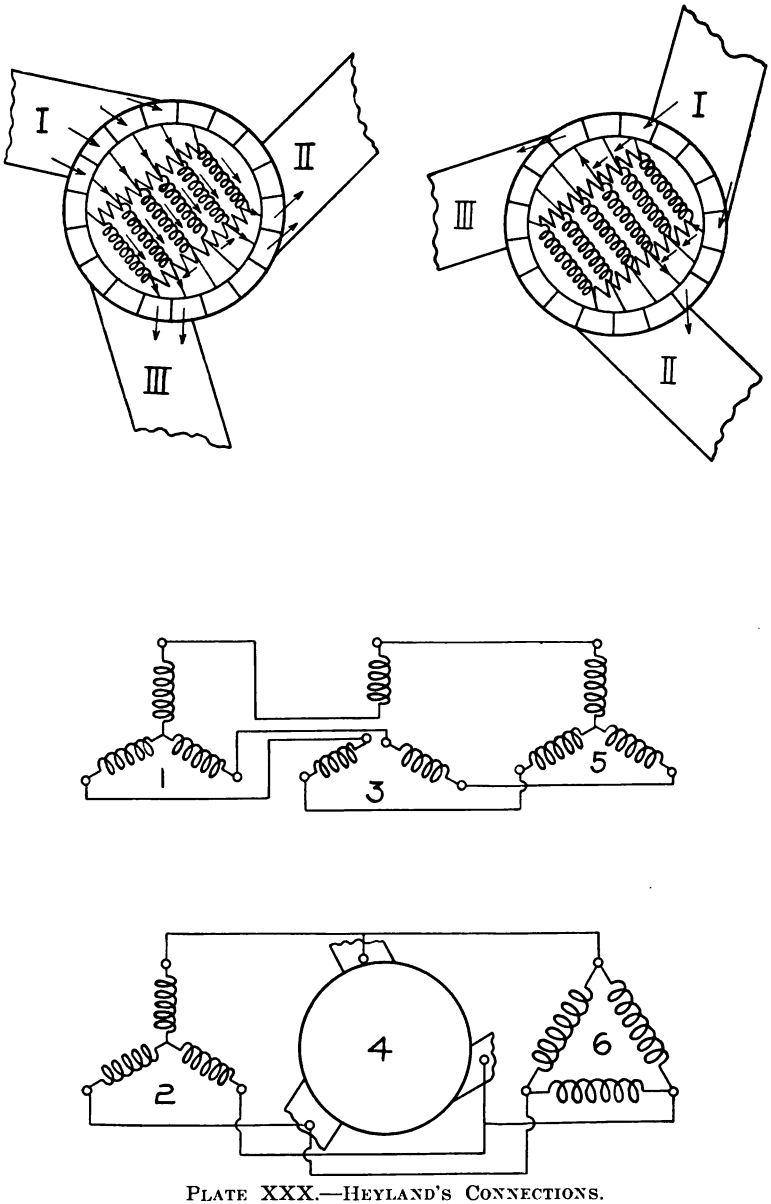



PLATE XXX.—HEYLAND'S CONNECTIONS.



Digitized by Google

belongs when it partakes, as so many do, so very strongly of the characteristics of both. This commutator has the same function of sifting out watt from wattless as in the induction machine pure and simple and the method of compounding is identical.

At first, for compounding, Heyland used a second set of three brushes placed on the commutator 90° ahead of the first set (since there is a 90° lag of wattless current behind e. m. f.). This worked very satisfactorily but the six brushes looked formidable so he reduced them to three by using (as is necessary with large machines) transformers and connecting (as shown in lower figure) the secondaries of the exciting transformers in mesh whereas the primaries are in star. This gives an additional angle of 30°, which added to the 90° gives 120°, thus coinciding with the next brush and making but three requisite.

In the lower figure, No. 1 is the primary side of the compounding transformer and No. 2, the secondary. No. 3 is the generator armature (stationary) and No. 4 is armature and commutator as shown in upper figure, while No. 5 is primary and No. 6 secondary of the exciting transformer.

By adjusting the number of turns in the compounding transformer any desired degree of compounding may be obtained while the main exciting current is varied if desired by variable resistance, although the idea is to fix the adjustments once and then leave them alone unless it is desired to correct for greater line drop, etc.



Digitized by Google

# BIBLIOGRAPHY

## Articles

*Elektrotechnische Zeitschrift:*

Thomälen, A. November 26, 1903; August 11, 1904.
Ossanna. August 2 , 1900.
Heyland, A. Page 561, 1894: page 632, 1896; August 8, 1901; August 21, 1901; November 21, 1901; December 12, 1901; January 9, 1902; June 26, 1902; January 22, 1903; January 29, 1903; February 5, 1903; March 19, 1903; May 28, 1903; June 26, 1903; July 23, 1903; November 5, 1903; December 17, 1903; November 1, 1906; February 7, 1907; February 14, 1907.
Grob, H. January 24, 1901; June 2, 1904; June 9, 1904.
Kuhlmann, K. April 18, 1901; vol. 22, page 341.
Neithammer, Dr. F. Vol. 22, page 474.
Bragstad, and La Cour. Vol. 25, page 993.
Beneschke, G. Vol. 25, page 738; vol. 25, page 1401.
Kolben. November 8, 1903, et seq.
Weisshaar, O. November 14, 1901.
Osnos. October 16, 1902; November 27, 1902.
Bodensteiner. August 21, 1902.
Heubach, J. April 27, 1899; May 4, 1899. Page 73, 1900; page 97, 1900.
Müller, P. Vol. 25, page 173.
Behrend, B. A. Jauuary 30, 1896; January 24, 1901.
Behn Eschenburg, Dr. Page 178, 1894.
Behn Eschenburg, Dr. and Cohen. Page 284, 1894.
Steinmetz, C. P. Page 78, 1896.
Görges, M. April 9, 1903; December 25, 1891.
Latour, M. May 22, 1902; July 3, 1902.
Sievert, A. August 1, 1901.

*Zeitschrift für Elektrotechnik:*

Sumec, J. K. January 4, 1903; April 14, 1901; August 27, 1905.
Besig, F. July 10, 1904.
La Cour, J. November 1, 1903, et seq.
Breslaur, Dr. M. April 20, 1902.
"E. A." Page 346, 1902.
Ossanna. Page 223, 1899; page 236, 1899; page 248, 1899.
Puluj, J. Page 63, 1904; page 80, 1904.
Rosskopf, June 11, 1905.
Hobart, H. M. March 13, 1904.
Alexander, and Fleischman. May 3, 1903; May 10, 1903.
Sartori. Page 188, 1903.

*Electrical World and Engineer:*

Way, W. C. December 25, 1905.
Press, A. September 9, 1905.
McAllister, A. S. January 17, 1903; vol. 41, page 109.
Averett. Vol. 46, page 270.



Digitized by Google

*Electrical World and Engineer*—Continued.
Reyval. October 7, 1905.
La Tour, M. Vol. 45, page 97; vol. 41, page 442; vol. 41, page 479; February 1, 1902; March 14, 1903; March 21, 1903.
Heyland, A. Vol. 40, page 817.
Danielson, E. Vol. 21, page 44.
Alexanderson, E. F. May 20, 1905.
Specht. February 25, 1905.
Heyland, A. Feburary 23, 1907.

*American Electrician*:
McAllister, A. S. January, 1903; July, 1903; November, 1903. Vol. 15, page 534.

*Transactions, American Institute of Electrical Engineers:*
Alexanderson, E. F. January, 1906.
Langsdorff, A. S. September, 1905.
Garfield, A. S. May, 1903.
Adams, C. A. May, 1903.
Franklin, W. S. May, 1903.
Behrend, B. A. Vol. 21, page 497; vol. 21, page 517.
Perkins. Vol. 24, page 649.

*Sammlung Elektrotechnischer Vorträge (Volt):*
Heyland, A. Vol. 2, paper 2.
Niethammer, Dr. F. Vol. 2, paper 3.
Arnold, E. and La Cour, J. Vol. 3, paper 1; vol. 6, paper 3.
Heubach, J. Vol. 4, paper 1; vol. 6, paper 1.
Breslaur, Dr. M. Vol. 4, paper 4.
Railing, Dr. Ing. A. Vol. 4, paper 6.

*Electrician (London):*
Butt, T. P. E. August 25, 1905.
Heyland, A. Vol. 36, page 505; vol. 36, page 578; vol. 36, page 651; vol. 36, page 719; vol. 36, page 753; vol. 53, page 509; January 24, 1902; October 26, 1906.
Breslaur, Dr. M. Vol. 51, page 803; vol. 51, page 903; vol. 51, page 940.
Eborall. Vol. 51, page 442; vol. 51, page 486.
Hawkins, C. C. January 27, 1899.
Ferraris, Prof. G. December 27, 1895.
(Translated entire from Turin Acad. Vol. 23, March, 1888. Rotazioni Elettrodynamische.)

*Electric Journal:*
Karapetoff. Vol. 1. ——.

*Western Electrician:*
Reist. Vol. 26, page 353.
Heyland, A. Vol. 37, page 144.

*Electrical Review (London):*
Parker, W. July 1, 1904.
Heyland, A. December 7, 1900.
Hooghwinkel, A. Vol. 53, page 122, et seq.

*Industry and Iron:*
Jackson, D. C. Vol. 21, page 124.

*Electricien:*
Allamet. Vol. 23, page 113.
La Tour, M. Vol. 23, page 164.

*Eclairage Electrique:*
Blondel. August 29, 1896; May, 1895; April 25, 1903; May 2, 1903. Vol. 35, page 121; vol. 35, page 167.
Lehmann, Dr. Th. August 22, 1903. Vol. 36, page 281; vol. 36, page 351.



Digitized by Google

*Eclairage Electrique*—Continued.
Neithammer, Dr. F. Vol. 40, p. 5. July, 1904.
Bethenod. Vol. 40, p. 253; vol. 40, p. 281.
Adams, C. A. Vol. 38, p. 23.
Osnos. Vol. 31, p. 279.
Hutin, and Leblanc. Vol. 40, p. 373.
Leblanc. August 29, 1896; October 26, 1901. Page 161, 1899; page 376, 1899.
La Tour, M. November 30, 1901; April 12, 1902.
Poincaire. January 18, 1902.
Heyland, A. March 22, 1902.

*Schweizerische Electrotechnische Zeitschrift:*
Lehmann, Dr. Th. Serials in 1904.
Niethammer, Dr. F. Serials in 1904.
Behn Eschenburg, Dr. Serials in 1904.

*Elektrotechnische Anzeiger:*
Zeigenberg. July 17, 1902; July 24, 1902; August 10, 1902; August 17, 1902; September 21, 1902; September 28, 1902; October 21, 1902; October 28, 1902.
Hahn. Page 678, 1904.

*Bulletin de la Societie International des Electriciens:*
Leblanc. Vol 15, p. 488.

*Industrie Electrique:*
Latour, H. B. Vol. 11, page 148; vol. 11, page 172; vol. 11, page 245; vol. 11, page 149.
Heyland, A. Vol. 11, page 244.

*Proceedings of the Institution of Electrical Engineers (London):*
Walker, S. F. April, 1905.

*Sibley Journal;*
McAllister, A. S. November, 1902.
Norris. March, 1901.

*Elektrotechnics und Maschinenbau:*
Lombardi. September 30, 1906.

*Electrical Engineer (London):*
Pensabene. February 12, 1904.
Davies. January 27, 1904.
Heyland, A. November 9, 1906.
Butt. Vol. 91, page 36.

*Proceedings of the International Electric Congress;*
La Tour. St. Louis, 1904.
Steinmetz, C. P. St. Louis, 1904.
Heyland, A. St. Louis, 1904.
Leblanc. Paris, 1900.

THESES UNIVERSITY OF WISCONSIN.

Whiting, M. A., *Induction Generator and Booster.* (For B. S.)
Foote, F. J.. *Design of Induction Motors.* (For E. E.)
Feige, H. *The Rotor Pressure Waves of Induction Motors at Synchronous Speed.* (For B. S.)
Scott, A. C. *Rotations Single-phase Alternator.* (For Ph. D.)



Digitized by Google

BOOKS.

McAllister, Dr. A. S., *Alternating Current Motors.*
Thompson, S. P., *Polyphase Electric Currents.*
Bedell, F., and Crehore, *Alternating Currents.*
Russell, A. S., *Alternating Currents* (2 vols.).
Raymond, E. B., *Alternating Current Engineering.*
Blondel, ——, and DuBois, *La Traction Electrique* (vol. 2).
Hobart, H. M., *Electric Motors.*
Jackson, D. C., *Alternating Currents.*
Hay, A., *Alternating Currents.*
Oudin, M. A., *Standard Polyphase Apparatus and Systems.*
La Tour, H. B., *The Induction Motor.*
Behrend, B. A., *Induction Motor.*
Heubach, J., *Der Drehstrommotor.*
Feldmann, C., *Asynchrone Generatoren.*
Niethammer, Dr. F., *Moteurs à Collecteur à Courants Alternatifs.*
Steinmetz, C. P., *Alternating Current Phenomena.*
Benischke, Dr. G., *Die Asynchronen Drehstrommotoren.*
Tesla, N., *Patents of.*
Kapp, G., *Transformers.*
Emde, F., *Die Arbeitsweise der Wechselstrommaschinen.*
Roessler, *Elektromotoren für Wechsel—und Dreh-strom.*



Digitized by Google

VOLUME I

*(Complete in ten numbers, with title-page, table of contents, and index.)*

No. 1. Track, by Leonor Fresnel Loree. 1894. 24 p. 25 cents. *Out of print.*

No. 2. Some practical hints in dynamo design, by Gilbert Wilkes. 1894. 16 p. 25 cents. *Out of print.*

No. 3. The steel construction of buildings, by Corydon Tyler Purdy. 1894. 27 p. 25 cents. *Out of print.*

No. 4. The evolution of a switchboard, by Arthur Vaughan Abbott. 1894. 32 p. 4 pl. 35 cents. *Out of print.*

No. 5. An experimental study of field methods which will insure to stadia measurements greatly increased accuracy, by Leonard Sewell Smith. 1895. 45 p. 1 pl. 35 cents. *Out of print.*

No. 6. Railway signaling, by William McCollough Grafton. 1895. 38 p. 35 cents.

No. 7. Emergencies in railroad work, by Leonor Fresnel Loree. 1895. 42 p. 35 cents.

No. 8. Electrical engineering in modern central stations, by Louis Aloysius Ferguson. 1896. 33 p. 35 cents.

No. 9. The problem of economical heat, light, and power supply for building blocks, school houses, dwellings, etc., by Gerdt Adolph Gerdtzen. 1896. 69 p. 45 cents.

No. 10. Topographical surveys, their methods and value, by John Lane Van Ornum. 1896. 39 p. 35 cents.

VOLUME II

*(Complete in ten numbers, with title-page, table of contents, and index.)*

No. 1. A complete test of modern American transformers of moderate capacities, by Arthur Hillyer Ford. 1896. 88 p. 35 cents.

No. 2. A comparative test of steam injectors, by George Henry Trautmann. 1897. 34 p. 25 cents.

No. 3. The superintendent of bridges and buildings, by Onward Bates. 1898. 30 p. 25 cents.

No. 4. Some unrecognized functions of our state universities, by John Butler Johnson. 1899. 20 p. 25 cents. *Out of print.*

No. 5. The transcontinental triangulation along the thirty-ninth parallel, by John Fillmore Hayford. 1900. 22 p. 6 pl. 25 cents.

No. 6. The chemical engineer, by Magnus Swenson. 1900. 13 p. 25 cents.

No. 7. Recently improved methods of sewage disposal, by John Butler Johnson. 1900. 19 p. 25 cents.

No. 8. An experimental study of the corrosion of iron under different conditions, by Carl Hambuechen. 1900. 47 p. 19 pl. 30 cents.

No. 9. The progress of the ceramic industry, by Edward Orton, Jr. 1903. 23 p. 25 cents.

No. 10. The effect of frequency on the steadiness of light emitted from an incandescent lamp, by Harold Seaman. 1903. 47 p. 25 cents.

VOLUME III

*(Complete in six numbers, with title-page and table of contents.)*

No. 1. An investigation of rotations produced by current from a single-phase alternator, by Arthur Curtis Scott. 1904. 162 p. 50 cents.

No. 2. The sources of water supply in Wisconsin, by William Gray Kirchoffer. 1905. 88 p. 50 cents.

No. 3. An investigation of the borides and the silicides, by Oliver Patterson Watts, 1906. 68 p. 30 cents.

No. 4. Tests on reinforced concrete beams, by Ernest Anthony Moritz. 1907. 76 p. 30 cents.

No. 5. A comparison of the effects of frequency on the light of incandescent and Nernst lamps, by Frederick William Huels. 1907. 51 p. 25 cents.

No. 6. Investigation of centrifugal pumps: Part I, A discussion of the theory of the centrifugal pump and tests of a six inch vertical cehtrifugal pump, by Clinton Brown Stewart. 1907. 141 p. 50 cents.

VOLUME IV

No. 1. Tests on plain and reinforced concrete, series of 1906, by Morton Owen Withey. 1907. pp. 66. 25 cents.

No. 2. Tests on plain and reinforced concrete, series of 1907, by Morton Owen Withey. 1907. pp. 69. 25 cents.

No. 3. An investigation of the hydraulic ram, by Leroy Francis Harza. 1908. pp. 110. 50 cents.

No. 4. Investigation of flow through large submerged orifices and tubes: Part I, Experiments with submerged tubes 4.0 feet square. By Clinton Brown Stewart. 1908. pp. 83. Price 25 cents.

No. 5. Current practice in steam engine design, by Ole N. Trooien, with foreword by Prof. H. J. Thorkelson. 1908. pp. 79. Price 25 cents.

No. 6. Self-excited polyphase asynchronous generators, by Lewis Fussell. 1908. pp. 87. Price 25 cents.

---

The Bulletin of the University of Wisconsin is published bimonthly at Madison. For postal purposes, all issues in all series of the Bulletin are included in one consecutive numbering as published, a numbering which has no relation whatever to the arrangement in series and volumes.

The Economics and Political Science series, the History series, the Philology and Literature series, the Science series, the Engineering series, and the University Extension series contain original papers by persons connected with the University. The series formerly issued as the Economics, Political Science, and History series was discontinued with the completion of the second volume and has been replaced by the Economics and Political Science series and the History series.

Persons who reside in the state of Wisconsin may obtain copies of the Bulletin free by applying to the Secretary of the Regents and paying the cost of transportation. No. 1 of Vol. 1 of the Economics, Political Science, and History series, Nos. 1 and 3 of Vol. 2 of the Philology and Literature series, No. 2 of Vol. 2 of the Science series, and Nos. 1–5 of Vol. 1 and No. 4 of Vol. 2 of the Engineering series are now out of print and can no longer be furnished. Bulletins issued since May 1, 1898, are entered as second-class mail matter and no charge is required by the University to cover cost of postage. The postage required for such of the earlier numbers as can now be furnished is as follows: Econ. ser., Vol. 1, No. 2, 8c; No. 3, 13c; Vol. 2, No. 1, 4c; Phil. ser., Vol. 1, No. 1, 5c; Sci. ser., Vol. 1, No. 1, 2c; No. 2, 2c; No. 3, 3c; No. 4, 3c; No. 5, 10c; Vol. 2, No. 1, 2c; Eng. ser., Vol. 1, No. 6, 2c; No. 7, 3c; No. 8, 2c; No. 9, 4c; No. 10, 3c; Vol. 2, No. 1, 4c; No. 2, 2c.

Any number of the Bulletin now in print will be sent postpaid to persons not residents of Wisconsin from the office of the Secretary of the Regents on receipt of the price. Title pages, tables of contents, and indexes to the first two volumes of all series, except the new Economics and Political Science series and the new History series, have been issued and will be furnished without cost on application to the University Librarian. Communications having reference to an exchange of publications should be addressed to the Librarian of the University of Wisconsin, Madison, Wis.

Digitized by Google

Digitized by Google

89010845774

b89010845774a

Digitized by Google

Digitized by Google

8910845774
b89010845774a

Digitized by Google